Dwpt Based FFT and ITS Application to SNR Estimation in OFDM Systems

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ABSTRACT

In this paper, wavelet packet (WP) based FFT and its application to SNR estimation is proposed. OFDM systems demodulate data using FFT. The proposed solution computes the exact FFT using WP and its computational complexity is of the same order as FFT, i.e. $O\ (N \log_2 N)$. SNR estimation is done inside wavelet packet based FFT block unlike other SNR estimators which perform SNR estimation after FFT. The data, so analyzed using wavelet packets, is used to perform SNR estimation in colored noise. The proposed estimator takes into consideration the different noise power levels of the colored noise over the OFDM sub-carriers. The OFDM band is divided into several sub-bands using wavelet packet and noise in each sub-band is considered white. The second-order statistics of the transmitted OFDM preamble are calculated in each sub-band and the power of the noise is estimated. The proposed estimator is compared with Reddy's estimator for colored noise in terms of mean squared error (MSE).

Keywords: SNR, Noise power estimation, Wavelet Packet, Adaptive modulation, OFDM.

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1. INTRODUCTION

Fourth Generation wireless and mobile systems characterized by broadband wireless systems are currently the focus of research and development among the researchers everywhere. The broadband wireless systems favor the use of orthogonal frequency division multiplexing (OFDM) modulation that allows high data-rate communication. A major advantage of OFDM systems is its ability to divide the input high rate data stream into many low-rate streams that are transmitted in parallel. Doing so increases the symbol duration and reduces the intersymbol interference over frequency-selective fading channels. This and other features of equivalent importance have motivated the adoption of OFDM as a standard for several applications such as digital video broadcasting (DVB) and broadband indoor wireless local area networks, broadband wireless metropolitan area networks and many others.

In order to exploit all these advantages and maximize the performance of OFDM systems; channel state information (CSI) plays a very important role. Signal-to-noise ratio (SNR) is a quantity that gives a comprehensive measure of CSI for each frame. An on-line SNR estimator thus provides the knowledge to decide whether a transition to higher bit rates would be favorable or not.

Signal-to-noise ratio (SNR) is defined as the ratio of the desired signal power to the noise power. Noise variance, and hence, SNR estimates of the received signal are very important parameters for the channel quality control in communication systems [1]. The search for a good SNR estimation technique is motivated by the fact that various algorithms require knowledge of the SNR for optimal performance. For instance, in OFDM systems, SNR estimation is used for power control, adaptive coding and modulation, turbo decoding etc.

SNR estimation indicates the reliability of the link between the transmitter and receiver. In adaptive system, SNR estimation is commonly used for measuring the quality of the channel and accordingly changing the system parameters. For example, if the measured channel quality is low, the transmitter may add some redundancy or complexity to the information bits (more powerful coding), or reduce the modulation level (better Euclidean distance), or increase the spreading rate (longer spreading code) for lower data rate transmission. Therefore, instead of implementing fixed information rate for all levels of channel quality, variable rates of information transfer can be used to maximize system resource utilization with high quality of user experience [2].

Many SNR estimation algorithms have been suggested in the last ten years [1]-[6] and also successfully implemented in OFDM systems using the pilot symbols [1,2,5]. Extracting pilots and using them for SNR estimation is computationally complex. The essential requirement for an SNR estimator in OFDM system is of low computational load. This is in order to minimize hardware complexity as well as the computational time. This is the motivating factor for the pursuit of an SNR estimator that does not require the manipulation of pilot symbols. SNR estimator, presented in the past, performed SNR estimation using pilot symbols at the back-end of the receiver in the OFDM system [2]. The SNR estimator presented in this paper performs SNR estimation inside FFT and makes use of only one OFDM synchronization preamble. In many SNR estimation techniques, noise is assumed to be uncorrelated or white. But, in wireless communication systems, where noise is mainly caused by a strong interferer, noise is colored in nature. According to best knowledge of authors, so far only Reddy's estimator considered colored noise scenario. Hence we use his work for the purpose of comparison in this paper. Reddy's estimator makes use of several OFDM symbols to perform SNR estimation; hence the computational complexity of finding good estimates is very high. So there is enough interest to design an efficient SNR estimator that is computationally simple.

OFDM demodulation uses discrete Fourier transform (DFT). An FFT (fast Fourier transform) is used to demodulate data. In this paper a novel wavelet packet based FFT and its application to SNR estimation is presented. The proposed solution computes the exact result, and its computational complexity is same order of FFT, i.e. $O\,(N\log_2 N)$.

The proposed SNR technique performs SNR estimation inside FFT unlike previous SNR estimators. SNR estimator for the colored noise in OFDM system is proposed. The algorithm is based on the two identical halves property of time synchronization preamble used in some OFDM systems. The OFDM band is divided into several sub-bands using wavelet packet and noise in each sub-band is considered white. The second-order statistics of the transmitted OFDM preamble are calculated in each sub-band and the power noise is estimated. Therefore, the proposed approach estimates both local (within smaller sets of subcarriers) and global (over all sub-carriers) SNR values. The short term local estimates calculate the noise power variation across OFDM sub-carriers. When the noise is white, the proposed algorithm works as good as the conventional noise power estimation schemes, showing the generality of the proposed method.

The remainder of the paper is organized as follows. In Section 2, the proposed wavelet packet based FFT technique is presented. Section 3 provides the proposed SNR estimation. Section 4 describes the Reddy' SNR estimator used for comparison with proposed SNR estimator. Section 5 presents simulation results and discussion. Section 6 concludes the paper**.**

2. PROPOSED WAVELET PACKET BASED FFT (DWPT-FFT)

The fundamental principle that the FFT relies on is that of decomposing the computation of the discrete Fourier transform of a sequence of length N into successively smaller discrete Fourier transforms of the even and odd parts. In the proposed method the even – odd separation is replaced by wavelet packet decomposition.

The block diagram of proposed DWPT-FFT is shown in Figure 1. The idea is borrowed from Guo [7]. Wavelet packet based FFT first performs Wavelet Packet decomposition, followed by reduced size FFT and butterfly operation as shown in Figure1. This can be extended so that WP analysis is 3 to 4 level analysis and FFT is $N/4$, $N/8$ or $N/16$ size FFT. Butterfly is appropriately designed.

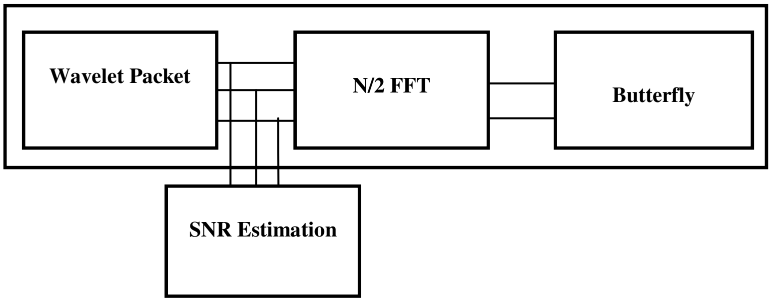
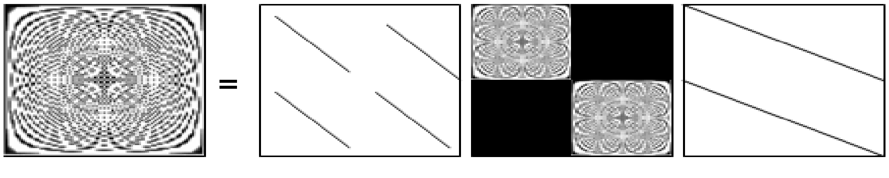


Figure 1: Block diagram of DWPT-FFT

The wavelet packet based FFT (DWPT-FFT) is represented by equation 1:

$$F_N = \begin{bmatrix} A_{N/2} & B_{N/2} \\ C_{N/2} & D_{N/2} \end{bmatrix} \begin{bmatrix} F_{N/2} & 0 \\ 0 & F_{N/2} \end{bmatrix} \left[WP_N \right] \tag{1}$$

where $A_{N/2}$,$B_{N/2}$,$C_{N/2}$ and $D_{N/2}$ are all diagonal matrices. In equation 1, the values on the diagonal of $A_{N/2}$ and $C_{N/2}$ are the length-N DFT of '***h***', the scaling filter coefficients, and the values on the diagonal of $B_{N/2}$ and $D_{N/2}$ are the length-N DFT of '***g***', the wavelet filter coefficients. The factorization can be visualized as



where we image the real part of DFT matrices, and the magnitude of the matrices for butterfly operations and the one-scale DWPT using *db3* wavelets. Clearly we can see that the twiddle factors have non-unit magnitude.

The above factorization suggests a DWPT-FFT algorithm. The block Diagram of length 8 algorithm is shown in Figure 2. Following this, the high pass and the low pass DWPT outputs go through separate length-4 DFT, then they are combined with butterfly operations.

Same procedure in Figure2 is iteratively applied to short length DFTs to get the full DWPT based FFT algorithm where the twiddle factors are the frequency wavelet filters. The detail of butterfly operations is shown in Figure 3 where '***i***' belongs to {*0,1,….N/2-1*}.

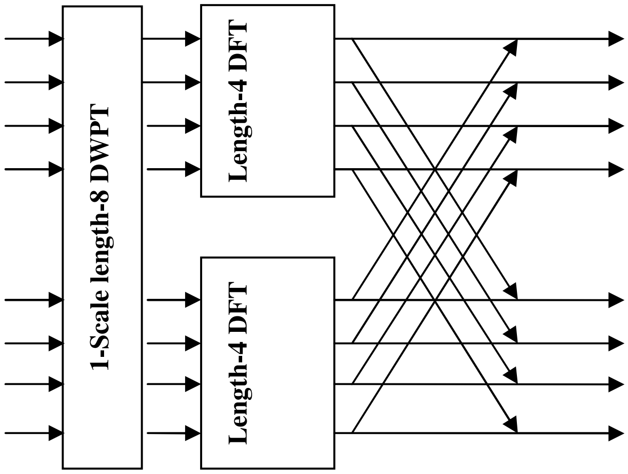


Figure 2: Last stage of length 8 DWPT-FFT

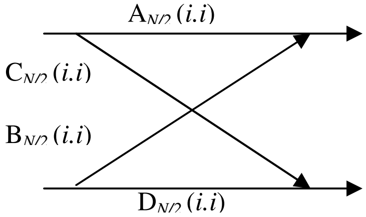


Figure 3: Butterfly operation in DWPT-FFT

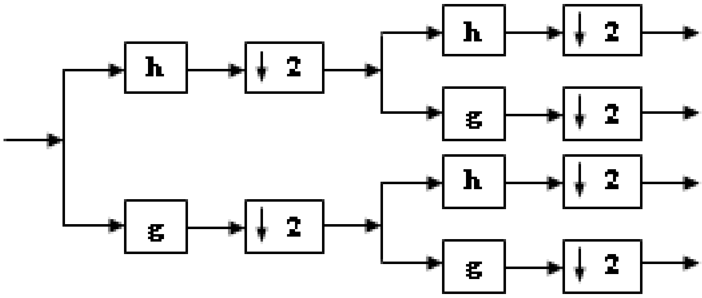


Figure 4: Two-scale discrete wavelet packet transforms

Wavelet packet allows a finer and adjustable resolution of frequencies at high frequency. Input data are first filtered by pair of filters **h** and **g** (low pass and high pass respectively) and then down sampled. The same analysis is further iterated on both low and high frequency bands as shown in Figure 4.

For the DWPT based FFT algorithm, the computational complexity is also $O\ (N\ log2N)$. However, the constant appears before $N\ log_2 N$ depends on the wavelet filters used.

3. SNR ESTIMATION

Wavelet Packet analyzed data becomes available for SNR estimation inside FFT block as shown in figure1. The synchronization OFDM preamble - the preamble which has two identical halves property as shown in figure 5, is obtained by loading constellation points with a PN sequence (P_{seq}) at even sub-carriers using equation 2 [9]:

$$P_{even}(k) = \begin{cases} \sqrt{2}.P_{seq}(k) & k = 2m \\ 0 & k = 2m+1 \end{cases} \qquad m = 1,2,3....n \tag{2}$$

where the factor of $\sqrt{2}$ is related to the 3 dB boost and 'k' shows the sub-carriers index. OFDM training/synchronization data of length 'N' is sent from the transmitter (T_x). To avoid intersymbol interference (ISI), a cyclic prefix (*CP*) is added.

After removing cyclic prefix at receiver, OFDM data is divided into 2^n sub-bands using periodic wavelet packets where 'n' shows the number of levels. The length of each sub-band is $N_{sub} = N/2^n$. It inherits the two identical halves property of synchronization preamble. The noise in each sub-band is considered white. The system's parameters and the structure of wavelet packet used for the simulations are calculated using [9] as shown in table 1.

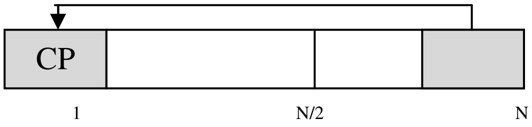


Figure 5: OFDM training symbol with cyclic prefix.

3.1. SIGNAL POWER AND NOISE POWER ESTIMATION

The received signal in the kth sub band is $r_k(n)$ and it is expressed as,

$$\begin{aligned} r_k(n) &= s'_k(n) + n_k(n) \\ &= s_k(n) * h_k(n) + n_k(n) \end{aligned} \tag{3}$$

The autocorrelation function of the received signal in k_{th} sub-band, $R^k_{rr}(m)$ has the following relationship to the autocorrelation of the transmitted sub-band signal, $R^k_{s's'}(m)$ and the noise, $R^k_{nn}(m)$:

$$R^k_{rr}(m) = R^k_{s's'}(m) + R^k_{nn}(m) \tag{4}$$

where $R^{k}_{s's'}(m) = \alpha_k R_{ss}(m)$, α_k is the attenuated channel power in the k^{th} sub-band. Over a small bandwidth of k^{th} sub band, it is safe to assume that, even in frequency selective channels, the attenuation is constant and equal to α_k .

The noise in channel is modeled as additive white Gaussian noise, thus its autocorrelation function can be expressed as

$$R_{nn}(m) = \sigma^{2}\delta(m) \tag{5}$$

where δ (m) is the discreet delta sequence and σ^2 is the power of noise.

A study of the OFDM signal shows that, as all the sub-carriers are present with equal power over the signal bandwidth, the power spectrum of an OFDM signal is nearly white and hence its autocorrelation is also given by

$$R_{s's'}(m) = \alpha_k \delta(m) \tag{6}$$

Hence, at zero lag, the autocorrelation contains both the signal power estimate and noise power estimate indistinguishable from each other.

However, because of the identical halves nature of the preamble the autocorrelation also has peaks where one half matches with other half on both sides of the zero delay. The autocorrelation of the transmitted and received scale 5 sub-band signal at SNR = 7 dB are shown in figure 6(a) and figure 6(b), respectively. It is clear that the autocorrelation values apart from the zero-offset are unaffected by the AWGN, so one can find the signal and noise powers from the zero-lag autocorrelation value '*L*'.

Taking into consideration the autocorrelation values for $L-N_{sub}/2$ and $L+N_{sub}/2$ signal power is given as

$$\overset{\wedge}{\alpha} = 2R_{rr}(L - N_{sub}/2) \tag{7}$$

Or

$$\overset{\wedge}{\alpha} = 2R_{rr}(L + N_{sub}/2) \tag{8}$$

Having obtained the power of signal in certain sub-band, noise power can be calculated using equation 8.

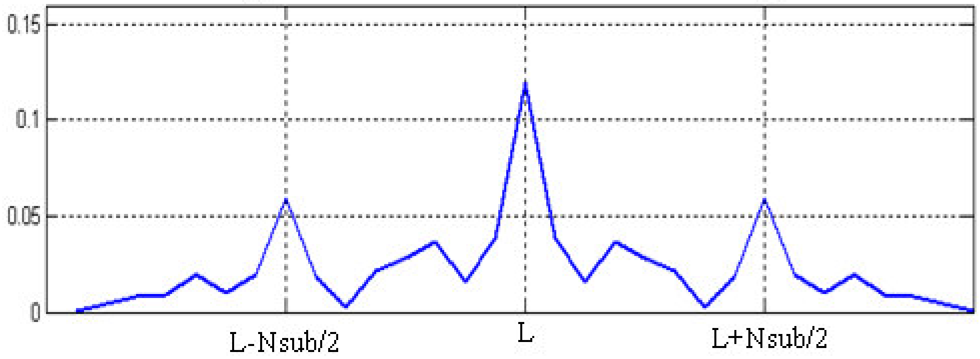
$$\hat{\sigma}^{2} = R_{rr}(L_{sub}) - \overset{\wedge}{\alpha} \tag{9}$$

Finally we can find the SNR estimates in the sub-band by using equation (7 or 8) and equation (9):

$$S\hat{N}R = \frac{\overset{\wedge}{\alpha}}{\hat{\sigma}^{2}} \tag{10}$$

where $S\hat{N}R$ is the estimated value for SNR.

(a) Autocorrelation of transmitted 5th band signal



(b) Autocorrelation of recieved 5th band signal

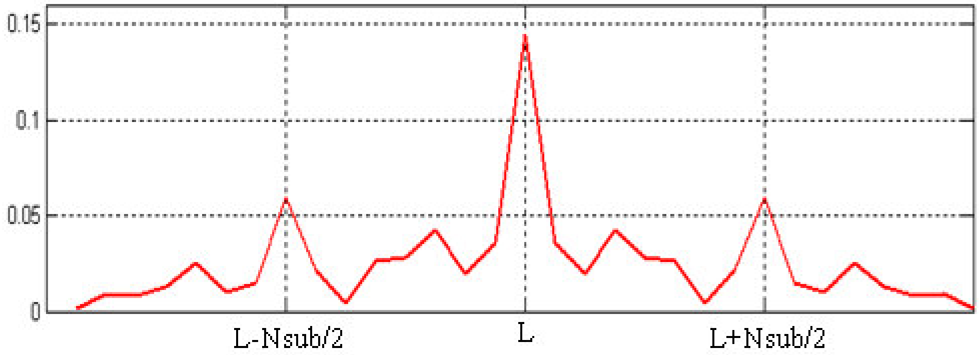


Figure 6: Autocorrelation plot of transmitted (a) and received (b) 5th band signal

4. REDDY'S SNR ESTIMATOR FOR COLORED NOISE

In this method channel estimation is performed in the first realization of the channel, using pilot symbols and this estimate is used to estimate the signal noise power. The suggested method can be used Additive White Gaussian Noise (AWGN) channel and for color dominated channel, in which the noise power varies across the frequency spectrum [2].

The system model is described in the frequency domain, where a signal is transmitted to obtain the estimated channel frequency response after which the instantaneous noise power mean square is determined. The transmitted signal includes white noise which is added by the channel of unknown amplitude. This is modeled in the frequency domain by the equation:

$$R_m(k) = S_m(k)H_m(k) + N_m(k) \tag{11}$$

where

$S_m(k)$ = Transmitted signal
$R_m(k)$ = Received signal
$N_m(k)$ = Channel white noise

The channel frequency response is estimated by transmitting preamble and performing division in the frequency domain of the received signal by the transmitted signal. When performing the division, the effect of noise is ignored. The pilot symbols are then used as the transmitted signal and the received signal in the pilot sub-carriers is used for the received signal and the estimated transfer function inserted in the equation to determine the noise power estimate. The noise power estimation is found by finding the difference between the noisy received signal and the noiseless signal.

$$\sigma^2_m(k) = \left|R_m(k) - \hat{S}_m(k)\,\hat{H}_m(k)\right|^2 \tag{12}$$

The difference between the actual channel frequency response and the estimated is the channel estimation error.

5. RESULTS AND DISCUSSIONS

In this section, we will first show that DWPT based FFT algorithm is able to compute FFT exactly. Following that, we will present the performance results of proposed estimator designed using wavelet packet based FFT. The results would include mean squared error of our estimation as compared to that of Reddy's.

The proposed DWPT-FFT computes the exact result. In order to show that the example of a chirp signal is taken and its FFT computed both from DWPT based algorithm and discrete Fourier transform (DFT) equation. The chirp signal is defined by $x(n) = e^{j\pi n^2} / N$, and its FFT as computed using DWPT and DFT are shown in figure 7.

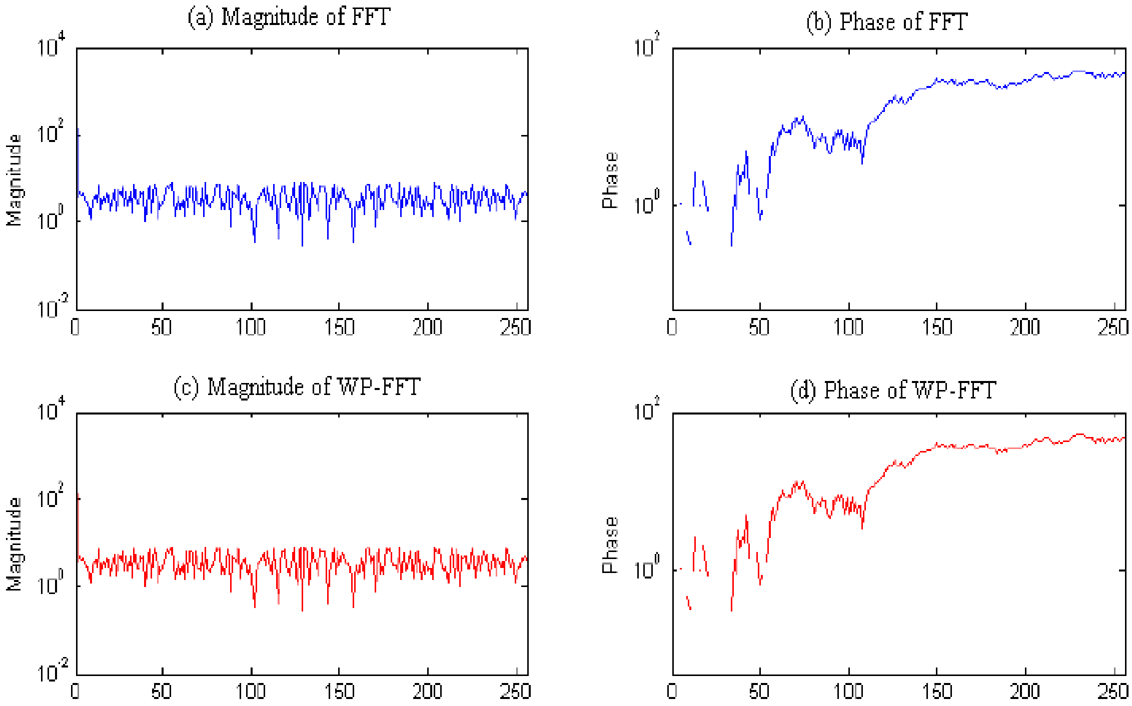


Figure 7: FFT result with and without wavelet packet

This performance of our estimator is measured in terms of Mean Squared Error. To obtain these results, OFDM system parameters given in Table 1 were chosen. The proposed SNR estimator is compared with Reddy's estimator for colored noise in OFDM systems. The parameters given in Table 1 are same for both estimators except that for Reddy's estimator 50 OFDM symbols are used to perform SNR estimation.

SNR is varied from 1 dB to 14 *dB* for each sub-band and the mean-squared error (MSE) is derived for the estimated SNR from 2000 trials according to the following formula

$$MSE = \frac{1}{2000}\sum_{i=1}^{2000}(\hat{SNR}(i) - SNR)^2 \tag{13}$$

From figure 8 and figure 9 it is clear that the proposed estimator gives better performance in SNR estimation as compared to Reddy estimator. It is observed that the proposed technique can estimate local statistics of the noise power when the noise is colored.

Table 1: Parameters for the simulation

Ifft size	256
Sampling Frequency = F_s	$20MHz.$
Sub Carrier Spacing= $\Delta f = F_s / Ifft$	1×10^{5}
Useful Symbol Time = $T_b = 1/\Delta f$	1×10^{-5}
CP Time =$T_g = G * T_b$ *where* $G = 1/4$	2.5×10^{-6}
OFDM Symbol Time = $T_s = T_b + T_g$	1.25×10^{-5}
$T_s = 5/4 * T_s$ *(Because ¼ CP makes the sampling faster by 5/4 time*	1.56×10^{-5}
$T_{sub} = T_s/16$	9.8×10^{-7}
Wavelet Packet Object Structure	
Wavelet Decomposition Command : wpdec *Size of initial data : [1 320]* *Order= 2* *Deptth=: 4* *Terminal nodes : [15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30]* -- *Wavelet Name : Daubechies (db3) ,* *Entropy Name : Shannon*	

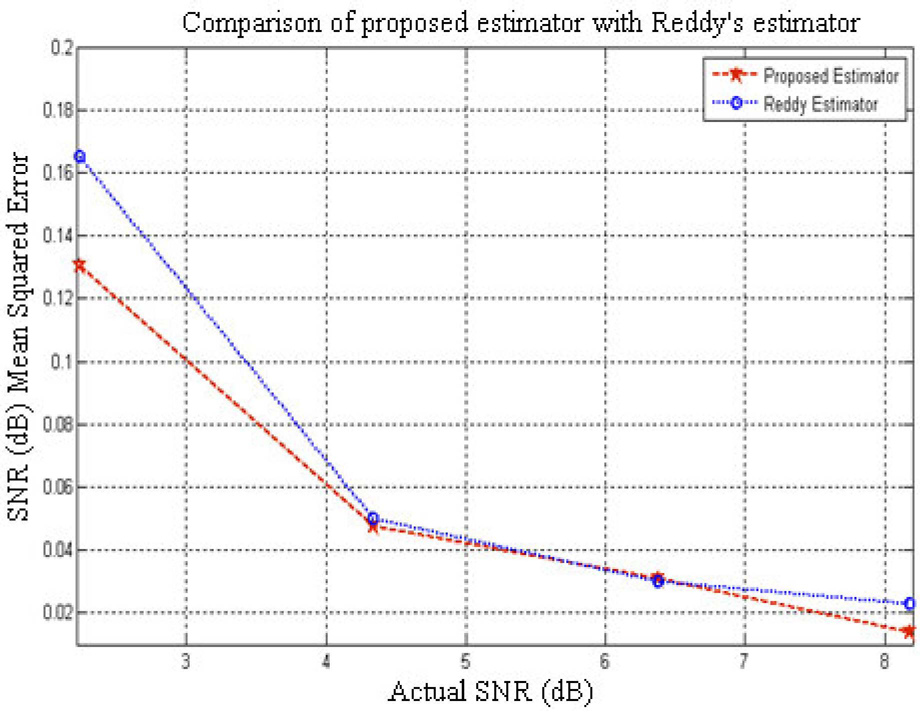


Figure 8: MSE performance of the proposed technique

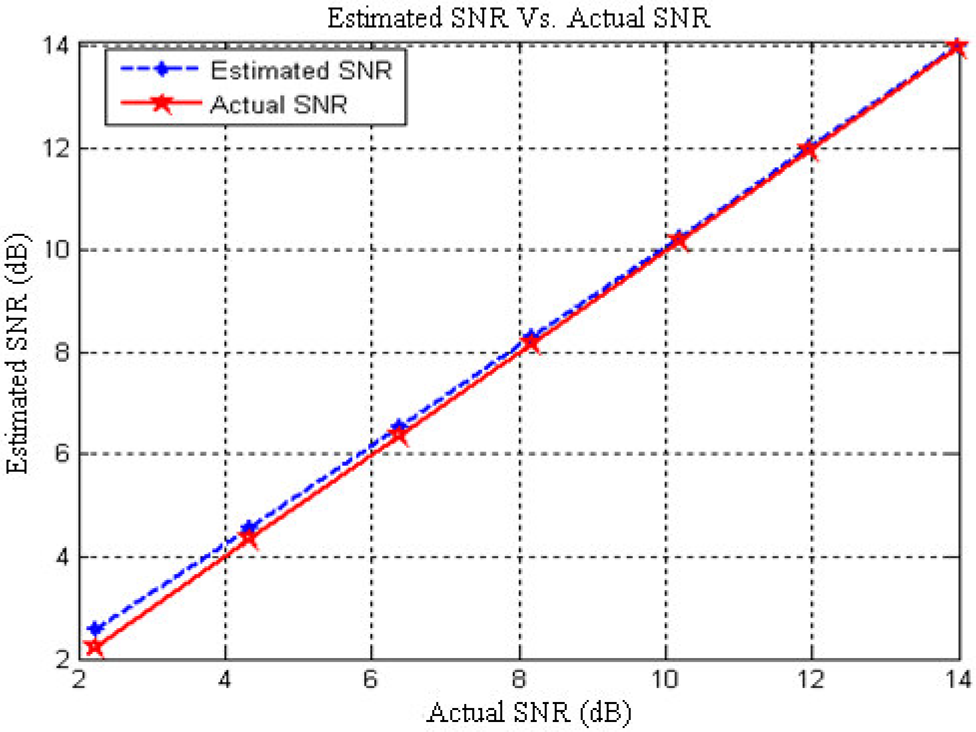


Figure 9: Actual SNR vs. Estimated SNR of colored noise

Complexity of proposed technique:

The proposed wavelet packet based FFT computes the exact result, and its computational complexity is of the same order of FFT, i.e. $O(\log_2 N)$.

The proposed autocorrelation based SNR estimator inside FFT makes use of one OFDM preamble so its complexity is $\sim 2N^3$ as compared to Reddy' estimator which have complexity $\sim 50N^3$ (where 50 shows the number of OFDM symbols used to get better SNR estimates after averaging over these OFDM symbols). The proposed estimator has relatively low computational complexity because it makes use of only one OFDM preamble signal to find the SNR estimates. The proposed estimator fulfills the criteria of a good SNR estimator because it is unbiased (i.e. it exhibits the smallest bias) and has the least variance of SNR estimates as seen from mean squared error diagrams..

6. CONCLUSION

In this paper, an algorithm to compute FFT using discrete wavelet packets is developed and applied to the problem of SNR estimation in OFDM systems inside FFT. Also, the technique is extended to SNR estimation under colored noise where the variation of the noise power across OFDM sub-carriers is allowed. The second-order statistics of the transmitted OFDM preamble are calculated in each sub-band and the noise power is estimated. Therefore, the proposed approach estimates both local (within smaller sets of subcarriers) and global (over all sub-carriers) SNR values. The short term local estimates calculate the noise power variation across OFDM sub - carriers. The results show that the proposed estimator performs better than other conventional methods. Complexity to find SNR estimates is much lower because our estimator makes use of only one OFDM preamble signal. This estimator fulfills the criteria of a good SNR estimator as it is unbiased and has the smallest variance of SNR estimates.

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