Probabilistic Analysis of a Desalination Plant with Major and Minor Failures and Shutdown During Winter Season

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Abstract

In many desalination plants, multi stage flash desalination process is normally used for sea water purification. The probabilistic analysis and profitability of such a complex system with standby support mechanism is of great importance to avoid huge loses. Thus, the aim of this paper is to present a probabilistic analysis of evaporators of a desalination plant with major and minor failure categories and estimating various reliability indicators. The desalination plant operates round the clock and during the normal operation; six of the seven evaporators are in operation for water production while one evaporator is always under scheduled maintenance and used as standby. The complete plant is shut down for about one month during winter season for annual maintenance. The water supply during shutdown period is maintained through ground water and storage system. Any major failure or annual maintenance brings the evaporator/plant to a complete halt and appropriate repair or maintenance is undertaken. Measures of plant effectiveness such as mean time to system failure, availability, expected busy period for maintenance, expected busy period for repair, expected busy period during shutdown & expected number of repairs are obtained by using semi-Markov processes and regenerative point techniques. Profit incurred to the system is also evaluated. Seven years real data from a desalination plant are used in this analysis.

Keywords: Desalination Plant, Minor/Major Failures, Repairs, Semi – Markov, Regenerative Process.

1. NOTATIONS

O Operative state of evaporator

U_{ms} Under Maintenance during summer

U_{mwb}	Under Maintenance during winter before service
U_{mwa}	Under Maintenance during winter after service
U_{msb}	Under Maintenance during summer before service
$F_{r_1 s}$	Failed unit is under minor repair during summer
$F_{R_1 S}$	Failed state of the evaporator due to minor repair during summer
$F_{r_2 s}$	Failed unit is under major repair during summer
$F_{R_2 S}$	Failed state of the evaporator due to major repair during summer
$F_{r_1 wb}$	Failed unit is under minor repair during winter before service
$F_{R_1 WB}$	Failed state of the evaporator due to minor repair during winter before service
$F_{r_2 wb}$	Failed unit is under major repair during winter before service
$F_{R_2 WB}$	Failed state of the evaporator due to major repair during winter before service
$F_{r_1 wa}$	Failed unit is under minor repair during winter after service
$F_{R_1 WA}$	Failed state of the evaporator due to minor repair during winter after service
$F_{r_2 wa}$	Failed unit is under major repair during winter after service
$F_{R_2 WA}$	Failed state of the evaporator due to major repair during winter after service
F	Failed state of one of the evaporator
β_1	Rate of the unit moving from summer to winter
β_2	Rate of the unit moving from winter to summer
λ	Rate of failure of any component of the unit
γ	Maintenance rate
γ_1	Rate of shutting down
γ_2	Rate of recovery after shut down during winter
α_1	Repair rate for minor repairs
α_2	Repair rate for major repairs
λ_1	Maintenance rate including the rate of inspection
p_1	Probability of occurrence of minor repair

p_2 Probability of occurrence of major repair

© Symbol for Laplace convolution

Ⓢ Symbol for Stieltje's convolution

* Symbol for Laplace Transforms

** Symbol for Laplace Stieltje's Transforms

C_0 Revenue per unit uptime

C_1 Cost per unit uptime for which the repairman is busy for maintenance

C_2 Cost per unit uptime for which the repairman is busy for repair

C_3 Cost per unit uptime for which the repairman is busy during shutdown

C_4 Cost per unit repair require replacement (all costs are taken in Rial Omani i.e., RO)

A_0 Steady state availability of the system

B_0^M Expected busy period of the repairman during maintenance

B_0^R Expected busy period of the repairman for repair

B_0^S Expected busy period of the repairman during shutdown

R_0 Expected number of repairs require replacement

$\emptyset_i(t)$ c.d.f. of first passage time from a regenerative state i to a failed state j

$q_{ij}(t)$, $Q_{ij}(t)$ p.d.f. and c.d.f. of first passage time from a regenerative state i to a regenerative state j or to a failed state j in (0, t]

$g_m(t)$, $G_m(t)$ p.d.f. and c.d.f. of maintenance rate

$g_{m1}(t)$,$G_{m1}(t)$ p.d.f. and c.d.f. of maintenance time including inspection

$g_s(t)$, $G_s(t)$ p.d.f. and c.d.f. of shutdown rate

$g_r(t)$, $G_r(t)$ p.d.f. and c.d.f. of recovery rate

$g_1(t)$, $G_1(t)$ p.d.f. and c.d.f. of repair rate for minor repairs

$g_2(t)$, $G_2(t)$ p.d.f. and c.d.f. of repair rate for major repairs

2. INTRODUCTION

Desalination is a water treatment process that removes the salt from sea water or brackish water. It is the only option in arid regions, since the rainfall is marginal. In many desalination plants, multi stage flash desalination process is normally used for water purification which is very expensive and involves sophisticated systems. Since, desalination plants are designed to fulfil the requirement of water supply for a larger sector in arid regions, they are normally kept in continuous production mode especially during summer except for emergency/forced/planned outages. It is therefore; very important that the efficiency and reliability of such a complex system is maintained in order to avoid big loses. Establishing the numerical results of various reliability indices are extremely helpful in understanding the significance of these failures/maintenances on plant performance and assesses the impact of these failures on the overall profitability of the plant.

Many researchers have expended a great deal of efforts in analysing industrial systems to achieve the reliability results that are useful for effective equipment/plant maintenance. Bhupender and Taneja [1] analysed a PLC hot standby system based on master-slave concept and two types of repair facilities, and many such analyses could be seen in the references therein. Mathew et al. [2] have presented an analysis of an identical two-unit parallel continuous casting plant system. Padmavathi et al. [3] have presented an analysis of the evaporator 7 of a seven unit desalination plant which fails due to any one of the six types of failures with the concept of inspection. Padmavathi et al. [4] explored a possibility of analyzing a desalination plant with emergency shutdown/unit tripping and online repair. Recently, Rizwan et al. [5] analyzed the desalination plant under the situation where repair or maintenance being carried out on a first come first served basis. Padmavathi et al. [6] analyzed a desalination plant with shutdown during winter season under the condition that the priority is given to repair over maintenance. However, analysis in [5] & [6] could have been better and more realistic results could be obtained if failures are categorized as minor and major failures and the repairs could be undertaken accordingly as the time to repair minor failures is comparatively lesser than the time required for fixing major failures. Also, it is viable to inspect the unit in order to identify the type of the failure and deal with it accordingly.

Thus, as a future direction of [5], a variation into the analysis is shown and hence this paper is an attempt to present a probabilistic analysis of the plant under minor and major failure categories including inspection for estimating various reliability indicators. Seven years failure data of a desalination plant in Oman have been used for this analysis. Component failure, maintenance, plant shutdown rates, and various maintenance costs involved are estimated from the data. The desalination plant operates round the clock and during the normal operation; six of the seven evaporators are in operation for water production while one evaporator is always under scheduled maintenance and used as standby evaporator. This ensures the continuous water production with minimum possible failures of the evaporators. The complete plant is shut down for about a month during winter season because of the low consumption of water for annual maintenance; the water supply during this period is maintained through ground water and storage system. The evaporator fails due to any one of the two types of failure viz., minor and major. Repairable and serviceable failures are categorised as minor failures, whereas the replaceable failures are categorised as major failures. Any major failure or annual maintenance brings the plant to a complete halt and goes under forced outage state.

Using the data, following values of rates and various costs are estimated:

- Estimated rate of failure of any component of the unit (λ) = 0.00002714 per hour
- Estimated rate of the unit moving from summer to winter (β_1) = 0.0002315 per hour
- Estimated rate of the unit moving from winter to summer (β_2) =0.0002315 per hour
- Estimated rate of maintenance (γ) = 0.0014881
- Estimated rate of shutting down (γ_1) = 0.000114155 per hour
- Estimated rate of recovery after shut down during winter (γ_2) = 0.0013889 per hour
- Estimated value of failure rate including inspection (λ_1) = 0.4013889 per hour
- Estimated value of repair rate of minor repairs (α_1) = 0.099216 per hour
- Estimated value of repair rate of major repairs (α_2) = 0.059701 per hour
- Probability of occurrence of minor repair (p_1) = 0.7419
- Probability of occurrence of major repair (p_2) = 0.2581
- The revenue per unit uptime (C_0) = RO 596.7 per hour

- Cost per unit uptime for which the repairman is busy for maintenance (C_1) = RO 0.0626 per hour
- The cost per unit uptime for which the repairman is busy for repair (C_2) = RO 0.003 per hour
- Cost per unit uptime for which the repairman is busy during shutdown (C_3) = RO 16.378 per hour
- The cost per unit repair require replacement (C_4) = RO 13.246 per hour

The plant is analysed probabilistically by using semi-Markov processes and regenerative point techniques. Measures of plant effectiveness/reliability indicators such as the mean time to system failure, plant availability, expected busy period during maintenance, expected busy period during repair, expected busy period during shut down and the expected number of repairs are estimated numerically.

3. MODEL DESCRIPTION AND ASSUMPTIONS

- There are seven evaporators in the desalination plant, of which 6 operate at any given time and one evaporator is always under scheduled maintenance.
- Maintenance of no evaporator is done if the repair of some other evaporator is going on.
- The various states of the evaporator are categorized under summer (states 0, 2, 5, 7,11and 12), winter (states 1, 4, 8, 9, 13 and 14), complete shutdown for overhaul/major service (state 3), and after major service (states 6, 10, 15, 16, 17and 18).
- The plant goes into shutdown for annual maintenance during winter season for one month.
- On completion of maintenance/repair, the repairman inspects as to whether the unit has failed due to minor/major failure, before putting the repaired unit into operation.
- If a unit is failed in one season, it gets repaired in that season only.
- Not more than two units fail at a time.
- During the maintenance of one unit, more than one of the other units cannot get failed.
- All failure times are assumed to have exponential distribution with failure rate (λ) whereas the repair times have general distributions.

4. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A state transition diagram showing the possible states of transition of the plant is shown in Fig. 1. The epochs of entry into states 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15 and 16 are regeneration points and hence these states are regenerative states. 11, 12, 13, 14, 17 and 18 are non-regenerative states. The transition probabilities are given by:

$$dQ_{00} = \gamma e^{-(6\lambda+\beta_1+\gamma)t}\,dt,\ \ dQ_{01} = \beta_1 e^{-(6\lambda+\beta_1)t}\,\overline{G_m}(t)dt,\ \ dQ_{02} = 6\lambda e^{-(6\lambda+\beta_1)t}\,\overline{G_m}(t)dt,$$
$$dQ_{11} = \gamma e^{-(6\lambda+\gamma_1+\gamma)t}\,dt,\ \ dQ_{13} = \gamma_1 e^{-(6\lambda+\gamma_1)t}\,\overline{G_m}(t)dt,\ \ dQ_{14} = 6\lambda e^{-(6\lambda+\gamma_1)t}\,\overline{G_m}(t)dt,$$
$$dQ_{24} = \beta_1 e^{-\beta_1 t}\,\overline{G_{m_1}}(t)dt,\ dQ_{25} = p_1 e^{-\beta_1 t}\,g_{m_1}(t)dt,\ \ dQ_{27} = p_2 e^{-\beta_1 t}\,g_{m_1}(t)dt,$$
$$dQ_{36} = \gamma_2 e^{-\gamma_2 t}\,dt$$
$$dQ_{43} = \gamma_1 e^{-\gamma_1 t}\,\overline{G_{m_1}}(t)dt,\ dQ_{48} = p_1 g_{m_1}(t)e^{-\gamma_1 t}\,dt,\ dQ_{49} = p_2 g_{m_1}(t)e^{-\gamma_1 t}\,dt,$$
$$dQ_{50} = e^{-6\lambda t}\,g_1(t)dt,\ \ dQ_{55}^{(11)} = \left(6\lambda e^{-6\lambda t} © 1\right)p_1 g_1(t)dt\,,dQ_{57}^{(11)} = \left(6\lambda e^{-6\lambda t} © 1\right)p_2 g_1(t)dt\,,$$
$$dQ_{60} = \beta_2\, e^{-(6\lambda+\beta_2)t}\,\overline{G_m}(t)dt,\ dQ_{66} = e^{-(6\lambda+\beta_2)t}\,g_m(t),\ dQ_{6,10} = 6\lambda\, e^{-(6\lambda+\beta_2)t}\,\overline{G_m}(t)dt,$$
$$dQ_{70} = e^{-6\lambda t}\,g_2(t)dt, dQ_{75}^{(12)} = \left(6\lambda e^{-6\lambda t} © 1\right)p_1 g_2(t)dt\,,dQ_{77}^{(12)} = \left(6\lambda e^{-6\lambda t} © 1\right)p_2 g_2(t)dt$$
$$dQ_{81} = e^{-(6\lambda+\gamma_1)t}\,g_1(t)dt,\qquad dQ_{83} = \gamma_1 e^{-(6\lambda+\gamma_1)t}\,\overline{G_1}(t)dt,$$
$$dQ_{83}^{(13)} = \left(6\lambda e^{-(6\lambda+\gamma_1)t} © \gamma_1 e^{-\gamma_1 t}\right)\overline{G_1}(t)dt\,,dQ_{88}^{(13)} = \left(6\lambda e^{-(6\lambda+\gamma_1)t} © e^{-\gamma_1 t}\right)p_1 g_1(t)dt\,,$$
$$dQ_{89}^{(13)} = \left(6\lambda e^{-(6\lambda+\gamma_1)t} © e^{-\gamma_1 t}\right)p_2 g_1(t)dt\,,$$
$$dQ_{91} = e^{-(6\lambda+\gamma_1)t}\,g_2(t)dt,\qquad dQ_{93} = \gamma_1 e^{-(6\lambda+\gamma_1)t}\,\overline{G_2}(t)dt,$$
$$dQ_{93}^{(14)} = \left(6\lambda e^{-(6\lambda+\gamma_1)t} © \gamma_1 e^{-\gamma_1 t}\right)\overline{G_2}(t)dt\,,dQ_{98}^{(14)} = \left(6\lambda e^{-(6\lambda+\gamma_1)t} © e^{-\gamma_1 t}\right)p_1 g_2(t)dt\,,$$

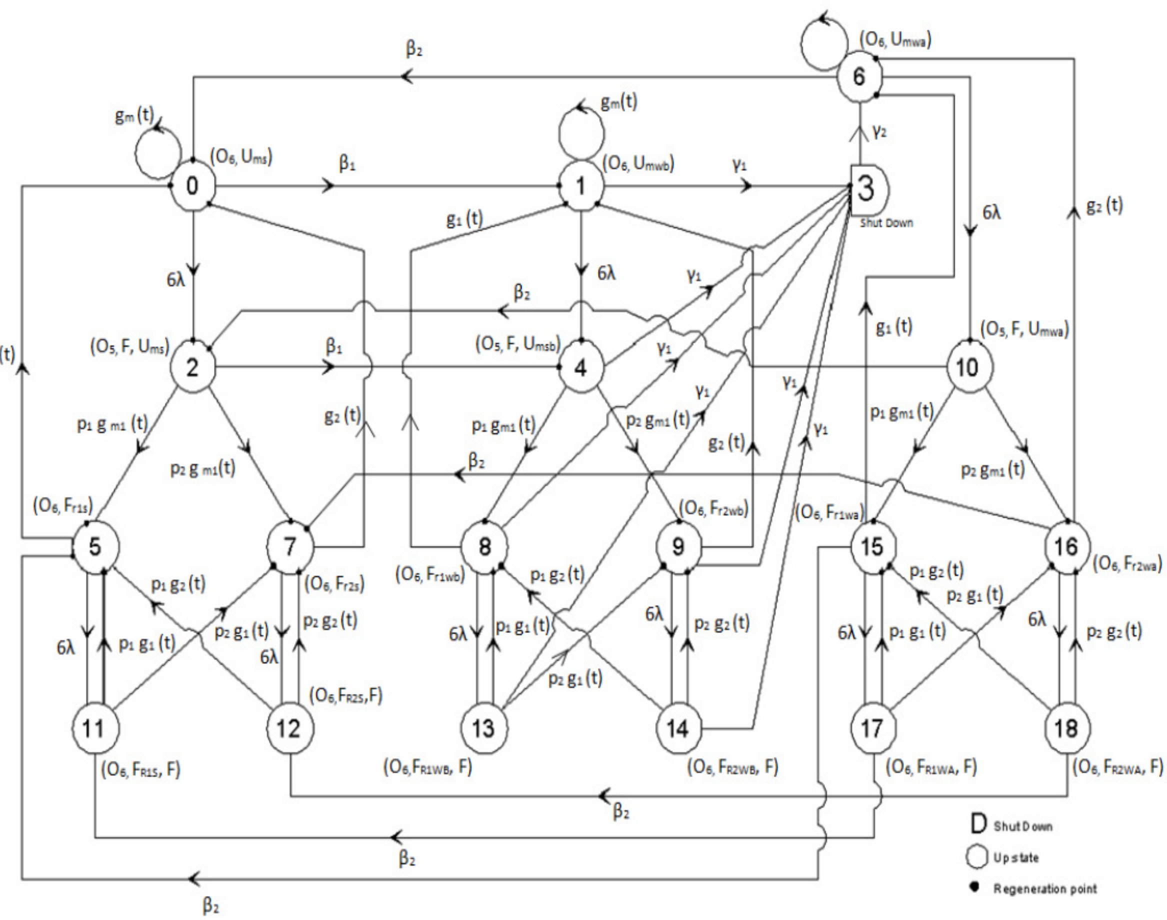

FIGURE 1: State Transition Diagram.

$$dQ_{99}^{(14)} = \left(6\lambda e^{-(6\lambda+\gamma_1)t} © e^{-\gamma_1 t}\right) p_2 g_2(t)dt,$$
$$dQ_{10,2} = \beta_2 e^{-\beta_2 t}\, \overline{G_{m_1}}(t)dt,\ dQ_{10,15} = p_1 e^{-\beta_2 t}\, g_{m_1}(t)dt,\ dQ_{10,16} = p_2 e^{-\beta_2 t}\, g_{m_1}(t)dt,$$
$$dQ_{15,5} = \beta_2\, e^{-(6\lambda+\beta_2)t}\, \overline{G_1}(t)dt,\ dQ_{15,6} = e^{-(6\lambda+\beta_2)t}\, g_1(t)dt,$$
$$dQ_{15,15}^{(17)} = \left(6\lambda e^{-(6\lambda+\beta_2)t} © e^{-\beta_2 t}\right) p_1 g_1(t)dt,\ dQ_{15,16}^{(17)} = \left(6\lambda e^{-(6\lambda+\beta_2)t} © e^{-\beta_2 t}\right) p_2 g_1(t)dt,$$
$$dQ_{15,5}^{(17,11)} = \left(6\lambda e^{-(6\lambda+\beta_2)t} © \beta_2\, e^{-\beta_2 t} ©1\right) p_1 g_1(t)dt,$$
$$dQ_{15,7}^{(17,11)} = \left(6\lambda e^{-(6\lambda+\beta_2)t} © \beta_2\, e^{-\beta_2 t} ©1\right) p_2 g_1(t)dt$$
$$dQ_{16,6} = e^{-(6\lambda+\beta_2)t}\, g_2(t)dt,\ dQ_{16,7} = \beta_2\, e^{-(6\lambda+\beta_2)t}\, \overline{G_2}(t)dt,$$
$$dQ_{16,16}^{(18)} = \left(6\lambda e^{-(6\lambda+\beta_2)t} © e^{-\beta_2 t}\right) p_2 g_2(t)dt,\ dQ_{16,15}^{(18)} = \left(6\lambda e^{-(6\lambda+\beta_2)t} © e^{-\beta_2 t}\right) p_1 g_2(t)dt,$$
$$dQ_{16,7}^{(18,12)} = \left(6\lambda e^{-(6\lambda+\beta_2)t} © \beta_2\, e^{-\beta_2 t} ©1\right) p_2 g_2(t)dt,$$
$$dQ_{16,5}^{(18,12)} = \left(6\lambda e^{-(6\lambda+\beta_2)t} © \beta_2\, e^{-\beta_2 t} ©1\right) p_1 g_2(t)dt$$

By these transition probabilities it can be verified that,

$$p_{00} + p_{01} + p_{02} = 1;\quad p_{11} + p_{13} + p_{14} = 1;\quad p_{24} + p_{25} + p_{27} = 1;\quad p_{36} = 1$$
$$p_{43} + p_{48} + p_{49} = 1;\quad p_{50} + p_{55}{}^{(11)} + p_{57}{}^{(11)} = 1;\ p_{60} + p_{66} + p_{6,10} = 1$$
$$p_{70} + p_{75}{}^{(12)} + p_{77}{}^{(12)} = 1;\quad p_{81} + p_{83} + p_{83}^{(13)} + p_{88}^{(13)} + p_{89}^{(13)} = 1$$
$$p_{91} + p_{93} + p_{93}^{(14)} + p_{98}^{(14)} + p_{99}^{(14)} = 1;\quad p_{10,2} + p_{10,15} + p_{10,16} = 1$$
$$p_{15,5} + p_{15,6} + p_{15,15}^{(17)} + p_{15,16}^{(17)} + p_{15,5}^{(17,11)} + p_{15,7}^{(17,11)} = 1$$
$$p_{16,6} + p_{16,7} + p_{16,16}^{(18)} + p_{16,5}^{(18,12)} + p_{16,7}^{(18,12)} + p_{16,15}^{(18)} = 1$$

The mean sojourn time (μ_i) in the regenerative state '*i*' is defined as the time of stay in that state before transition to any other state. If *T* denotes the sojourn time in the regenerative state '*i*', then:

$$\mu_i = E(T) = P(T > t);$$
$$\mu_0 = \frac{1}{(6\lambda + \beta_1 + \gamma)};\ \mu_1 = \frac{1}{(6\lambda + \gamma_1 + \gamma)};\ \mu_2 = \frac{1}{(\beta_1 + \lambda_1)};\ \mu_3 = \frac{1}{\gamma_2};$$
$$\mu_4 = \frac{1}{(\gamma_1 + \lambda_1)};\ \mu_5 = \frac{1}{(6\lambda + \alpha_1)};\ \mu_6 = \frac{1}{(6\lambda + \beta_2 + \gamma)};\ \mu_7 = \frac{1}{(6\lambda + \alpha_2)};$$
$$\mu_8 = \frac{1}{(6\lambda + \alpha_1 + \gamma_1)};\ \mu_9 = \frac{1}{(6\lambda + \alpha_2 + \gamma_1)};\ \mu_{10} = \frac{1}{(\beta_2 + \lambda_1)};$$
$$\mu_{15} = \frac{1}{(6\lambda + \beta_2 + \alpha_1)};\ \mu_{16} = \frac{1}{(6\lambda + \alpha_2 + \beta_2)}$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entry into state 'i' is mathematically stated as:

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -q_{ij}^{*\prime}(0),$$

Thus,

$$m_{00} + m_{01} + m_{02} = \mu_0;\ m_{11} + m_{13} + m_{14} = \mu_1;\ m_{24} + m_{25} + m_{27} = \mu_2;\ m_{36} = \mu_3$$
$$m_{43} + m_{48} + m_{49} = \mu_4;\ m_{50} + m_{55}^{(11)} + m_{57}^{(11)} = k_1\ (\text{say});\ m_{60} + m_{66} + m_{6,10} = \mu_6$$
$$m_{70} + m_{75}^{(12)} + m_{77}^{(12)} = k_2\ (\text{say});\ m_{81} + m_{83} + m_{83}^{(13)} + m_{88}^{(13)} + m_{89}^{(13)} = k_3\ (\text{say});$$
$$m_{91} + m_{93} + m_{93}^{(14)} + m_{98}^{(14)} + m_{99}^{(14)} = k_4\ (\text{say});\ m_{10,2} + m_{10,15} + m_{10,16} = \mu_{10}$$
$$m_{15,5} + m_{15,6} + m_{15,15}^{(17)} + m_{15,16}^{(17)} + m_{15,5}^{(17,11)} + m_{15,7}^{(17,11)} = k_5\ (\text{say});$$
$$m_{16,6} + m_{16,7} + m_{16,16}^{(18)} + m_{16,5}^{(18,12)} + m_{16,7}^{(18,12)} + m_{16,15}^{(18)} = k_6\ (\text{say})$$

5. THE MATHEMATICAL ANALYSIS

5.1 Mean Time to System Failure

To determine the mean time to system failure, the failed states are considered as absorbing states and applying the arguments used for regenerative processes, the following recursive relation for $\phi_i(t)$ is obtained:

$$\phi_0(t) = Q_{00}(t) Ⓢ \phi_0(t) + Q_{01}(t) Ⓢ \phi_1(t) + Q_{02}(t)$$
$$\phi_1(t) = Q_{11}(t) Ⓢ \phi_1(t) + Q_{13}(t) Ⓢ \phi_3(t) + Q_{14}(t)$$
$$\phi_3(t) = Q_{36}(t) Ⓢ \phi_6(t)$$
$$\phi_6(t) = Q_{60}(t) Ⓢ \phi_0(t) + Q_{66}(t) Ⓢ \phi_6(t) + Q_{6,10}(t)$$

Taking the Laplace Stieltje's transforms of the above equations and solving them for $\phi_0^{**}(s)$;

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)}$$

Where,

$$N(s) = Q_{02}^{**}(s) - Q_{02}^{**}(s)Q_{11}^{**}(s) - Q_{02}^{**}(s)Q_{66}^{**}(s) + Q_{02}^{**}(s)Q_{11}^{**}(s)Q_{66}^{**}(s) + Q_{01}^{**}(s)Q_{14}^{**}(s) - Q_{01}^{**}(s)Q_{14}^{**}(s)Q_{66}^{**}(s) + Q_{01}^{**}(s)Q_{13}^{**}(s)Q_{36}^{**}(s)Q_{6,10}^{**}(s)$$

$$D(s) = 1 - Q_{00}^{**}(s) - Q_{11}^{**}(s) - Q_{66}^{**}(s) + Q_{00}^{**}(s)Q_{11}^{**}(s) + Q_{00}^{**}(s)Q_{66}^{**}(s) + Q_{11}^{**}(s)Q_{66}^{**}(s) - Q_{00}^{**}(s)Q_{11}^{**}(s)Q_{66}^{**}(s) - Q_{01}^{**}(s)Q_{13}^{**}(s)Q_{36}^{**}(s)Q_{60}^{**}(s)$$

The mean time to system Failure (MTSF), when the unit started at the beginning of state 0 is given by:

$$\mathrm{MTSF} = \lim_{s\to 0} \frac{1-\emptyset_0^{**}(s)}{s} = \frac{N_1}{D}$$

where,

$N_1 = p_{11}\mu_1 - p_{11}p_{66}\mu_1 + p_{13}p_{36}p_{60}p_{01}\mu_1 - p_{11}p_{00}\mu_1 + p_{11}p_{66}p_{00}\mu_1 + p_{13}p_{36}p_{60}p_{01}\mu_3 + p_{66}\mu_6 - p_{11}p_{66}\mu_6 - p_{66}p_{00}\mu_6 + p_{11}p_{66}p_{00}\mu_6 + 2p_{13}p_{36}p_{60}p_{01}\mu_0 + p_{00}\mu_0 - p_{11}p_{00}\mu_0 - p_{66}p_{00}\mu_0 + p_{11}p_{66}p_{00}\mu_0 + p_{02}\mu_0 + p_{01}p_{14}\mu_0 + p_{01}p_{14}\mu_1 - p_{02}p_{11}\mu_0 - p_{02}p_{11}\mu_1 - p_{02}p_{66}\mu_0 - p_{02}p_{66}\mu_6 - p_{01}p_{14}p_{66}\mu_0 - p_{01}p_{14}p_{66}\mu_1 - p_{01}p_{14}p_{66}\mu_6 + p_{02}p_{11}p_{66}\mu_0 + p_{02}p_{11}p_{66}\mu_1 + p_{02}p_{11}p_{66}\mu_6 + p_{01}p_{13}p_{36}p_{69}\mu_0 + p_{01}p_{13}p_{36}p_{6,10}\mu_1 + p_{01}p_{13}p_{36}p_{69}\mu_3 + p_{01}p_{13}p_{36}p_{6,10}\mu_6$

$$D = 1 - p_{11} - p_{66} + p_{11}p_{66} - p_{13}p_{36}p_{60}p_{01} - p_{00} + p_{11}p_{00} + p_{66}p_{00} - p_{11}p_{66}p_{00}$$

Similarly, by employing the arguments used for regenerative processes, we obtain the recursive relations for other reliability indices; availability analysis of the plant, expected busy period for maintenance, expected busy period for repair, expected busy period during shut down, and the expected number of repairs. The profit incurred by the plant is also evaluated by incorporating the steady-state solutions of various reliability indices and costs:

$$P = C_0A_0 - C_1B_0^M - C_2B_0^R - C_3B_0^S - C_4R_0$$

6. PARTICULAR CASE

For this particular case, it is assumed that the failures are exponentially distributed whereas other rates are general. Using the values estimated from the data as summarized in section 1 and expressions of various reliability indicators as shown in section 4, the following values of the measures of system effectiveness/reliability indicators are obtained:

Mean Time to System Failure = 256 days

Availability (A_0) = 0.9603

Expected Busy period for Maintenance (B_0^M) = 0.9584

Expected Busy period for repair(B_0^R) = 0.0018

Expected Busy period during shutdown (B_0^S) = 0.0397

Expected number of repairs (R_0) = 0.0002

Profit (P) = RO 572.287 per unit uptime.

7. CONCLUSION

Measures of plant effectiveness in terms of reliability indices have been estimated numerically. Estimated reliability results facilitate the plant engineers in understanding the system behavior and thereby open a scope of improving the performance of the plant by adopting suitable maintenance strategies. As a future direction the modeling methodology could be extended for similar industrial complex system performance analysis.

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