

## Enhanced Spectral Reflectance Reconstruction Using Pseudo-Inverse Estimation Method

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### Abstract

This paper will present an enhanced approach for the reconstruction of spectral reflectance by the combination between two methods, the Pseudo-Inverse (PI) as the base formula, whilst adaptively selecting the training samples as performed in the Adaptive Wiener estimation method proposed by Shen and Xin for the estimation of the spectral reflectance. This enhancement will be referred to as Adaptive Pseudo-Inverse (API) through this research.

Training and verification datasets have been prepared from GretagMacbeth ColorChecker CC chart, Kodak Color Chart and a specially designed palette of Japanese organic and inorganic mineral pigments to test and compare the estimation results, using the Pseudo-Inverse and Adaptive Pseudo-Inverse method. The performance of spectral reconstruction methods will be presented in terms of spectral and colorimetric error for the estimation accuracy. The experimental results showed that the proposed method achieved better performance and noticeable decline in spectral estimation error.

**Keywords:** Adaptive Pseudo-Inverse, Spectral Reflectance Reconstruction, Colorimetry.

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### 1. INTRODUCTION

Some of the main aims of a multispectral system are the efficient extraction of spectral and colorimetric information, and in this regard several algorithms have been developed for best estimation of spectral reflectance and the reproduction of color information.

In this research we will focus on the estimation of spectral reflectance using Pseudo-Inverse (PI) method [1:3]. PI is known to be a traditional and straightforward method while it lack of accuracy and efficiency. PI estimates spectral reflectance without priori knowledge of acquisition system, depends on system responses and reflectance of training samples to get best transformation matrix aiming to minimize the spectral error between the actual reflectance measurement and the estimated one.

More accurate estimation can be extracted from other methods like Wiener estimation method [4:8] which depends on spectral responsivity, spectral reflectance and imaging noise. While the original calculations of Wiener depends on all available training samples, Shen and Xin proposed a modified approach of the original Wiener called Adaptive Wiener [9] which can estimate

spectral reflectance - without priori knowledge about the spectral characteristics of the verification sample - depending on the idea of adaptively selecting the training samples which had a strong impact on the color accuracy of the reflectance reconstruction.

This can be achieved by, firstly, selecting the closest responses of training samples to the verification sample, then calculating the weights of each and finally, recalculating the transformation matrix according to the selected training samples for the verification sample.

As in Shen and Xin approach we will adopt the adaptive selection of training samples to be integrated with Pseudo-Inverse method for the estimation of spectral reflectance which will be examined for estimation accuracy using some datasets.

## 2. FORMULATION OF MULTISPECTRAL IMAGING

The response of a digital camera can be formulated by this equation [9-10]:

$$V_i(x, y) = \int t_i(\lambda)E(\lambda)S(\lambda)r(x, y; \lambda)d\lambda + n_i(x, y) \quad (1)$$

$i = 1, \dots, m,$

Where  $V_i(x, y)$  is the response of the camera in  $(x, y)$  coordinate with  $i$ th color filters,  $t_i(\lambda)$  is the transmission of the  $i$ th filter,  $E(\lambda)$  is the spectral power distribution of the light,  $S(\lambda)$  is the sensitivity of the camera,  $r(x, y; \lambda)$  is the reflectance in  $(x, y)$  coordinates, and  $n_i$  is the additive noise for each channel which is ignored for simplicity and  $m$  is the number of channels.

$t_i(\lambda)$ ,  $E(\lambda)$  and  $S(\lambda)$  are the unknown factors where they are merged in  $F_i(\lambda)$  which is known as spectral responsivity.

Using vector-matrix notation, this equation can be written as follows:

$$V_{(mx1)} = F \cdot r_{(Kx1)} \quad (2)$$

For each pixel in the image,  $v$  is the vector of camera response and  $r$  is the vector of the reflectance spectrum.

The estimation of reflectance spectra can be obtained by

$$r_{est} = G \cdot V_{verification} \quad (3)$$

where  $v$  is the response of the camera and  $G$  is the estimation matrix which aims to reduce the minimum square error between original  $r$  and estimated  $r_{est}$  according to the used estimation method.

Using the traditional Pseudo-Inverse [1-3] to get the estimation matrix  $G_{PI}$

$$G_{PI} = r_{training} \cdot Pinv(V_{training}) \quad (4)$$

$$Pinv(V) = V^t \cdot (V \cdot V^t)^{-1}$$

$r_{training}$  and  $V_{training}$  are known from the measured training samples and  $Pinv(V_{training})$  is the Pseudo-Inverse of the camera response  $v_{training}$ . By solving Eq(4) and getting  $G_{PI}$  (Estimation Matrix based on Pseudo-Inverse method) we can substitute it in Eq(3) using  $V_{verification}$  as system response of verification sample to get the spectral reflectance  $r_{est}$  of this sample.

From this point in the research we will integrate the original Pseudo-Inverse with the adaptive approach of Shen and Xin [9]; after getting the estimated spectral reflectance  $r_{est}$  of the verification sample based on all training samples Eq(3), we start adaptively to select the training samples according to their spectral similarity to the  $r_{est}$  which is calculated from the following equation:

$$d_k = \alpha \text{mean} \left\{ \left| \frac{r_k}{\|r_k\|} - \frac{r_{est}}{\|r_{est}\|} \right| \right\} + (1 - \alpha) \text{max} \left\{ \left| \frac{r_k}{\|r_k\|} - \frac{r_{est}}{\|r_{est}\|} \right| \right\}, \quad (5)$$

Where  $d_k$  is the spectral similarity between each training sample and the verification sample,  $\alpha$  is a scaling factor ( $\alpha = 0.5$  in this research),  $|x|$  means the absolute values of elements in vector  $x$  and finally  $r_k$  is the measured spectral reflectance of each training sample. All reflectances are normalized from [0 - 1] and the mean and max spectral distances between two similar reflectances shouldn't be large.

By sorting the training samples in ascending order according to their spectral similarity and selecting  $L$  samples of it where  $d_1 < d_2 < \dots < d_L$  we get the  $L$  samples which are the more close samples to the verification sample.

Recalculating the Eq(4) and getting the new estimation matrix based on the selected  $L$  training sample as follows,

$$G_{API} = r_{training}(L) \cdot P_{inv} \left( V_{training}(L) \right) \quad (6)$$

Then substituting  $G_{API}$  in Eq(3) to get the new spectral reflectance of verification sample  $r_{est}$ .

### 3. EXPERIMENTAL

In this research, the multispectral imaging system comprises Niji Scanner [14] of Kyoto University (Advanced Imaging Technology Laboratory) with a monochrome line camera, led light system of known spectral power distribution and set of 8 band pass and sharp cut filters in the range of 420nm to 700nm. IR cut filter were used throughout the full scanning process which has been done in a dark room. The GretagMacbeth ColorChecker CC (24 patches), Kodak Color Chart (18 patches) and a specially designed palette of Japanese organic and inorganic mineral pigments (173 patches) [15] were collected and prepared to be used as the primary training and verification datasets. The reflectance of color targets were measured by X-Rite spectrophotometer [16] and re-sampled in the range of 400-700nm at 10nm interval. In this experiment, each target is tested separately for training and testing and the spectra has been reconstructed for each patch using different  $L$  (number of selected training samples) to see the effect of that on the color and estimation accuracy. Comparisons have been conducted between the PI and the API methods. The results of the comparison have been concluded according to the spectral and colorimetric error, using the mean squared error equation for spectral error between the actual and the estimated spectral reflectance and the equation of  $\Delta E_{00}$  obtained from the formula of CIEDE2000[11:13] under D65 as standard illuminant for colorimetric error.

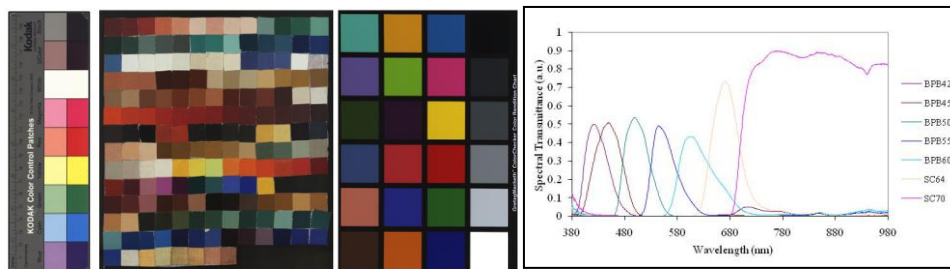
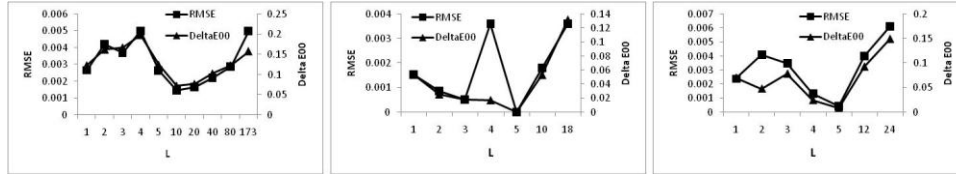


FIGURE 1: Color Targets Left and Spectral Transmittance of the Band Pass Filters Right.

Figure 1 shows the three color targets that have been used; The Kodak Color Chart, the Japanese pigments palette and the GretagMacbeth ColorChecker CC, in addition to the spectral transmittance of the 7 band pass filters Fujifilm BPB42, BPB45, BPB50, BPB55, BPB60, SC64, SC70 in the range from 380 to 980nm.

Each dataset has been investigated with different number of selected training samples (L) for Eq(6) as presented in Figure 2. In case of Kodak and Macbeth targets, which have less number of patches between 18 and 24, L value = 5 was the most appropriate number with the least spectral and colorimetric errors but in the Japanese palette case with 173 patches L value =10 was the point with the least errors for both spectral and colorimetric. In general datasets shows that both errors are changing in an increase monotonic trend with L.



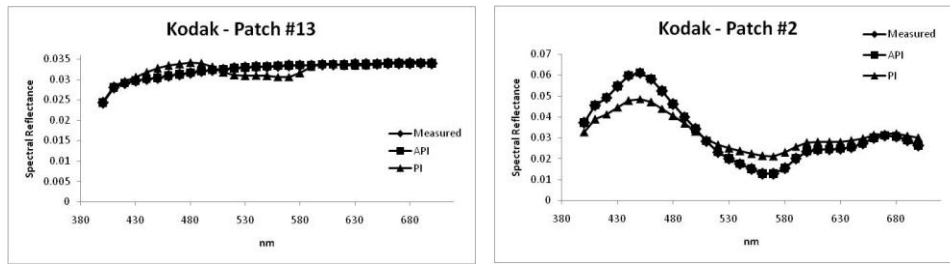
**FIGURE 2:** Japanese pigments palette (left) , Kodak chart (middle), Macbeth CC patches (right). The effect of using different L values on the spectral and colorimetric errors for the three datasets.

The spectral rms and colorimetric errors are shown in the Table1 that includes the mean, standard deviation and maximum of the tested methods for the three target datasets. Results of API method are presented according to the most appropriate L value. It is noticed that the results of the proposed method API is showing noticeable improvement over the PI method after applying the adaptive selection on the training samples. Except for the Japanese palette, although the rmse of the API method is less than the PI, the max value of the colorimetric error is showing higher value than the PI method, this is probably the influence of the duplicates of dark and shiny patches. Samples from best and worst cases for the spectral and colorimetric errors are presented in Figure (3a:3c) for each dataset.

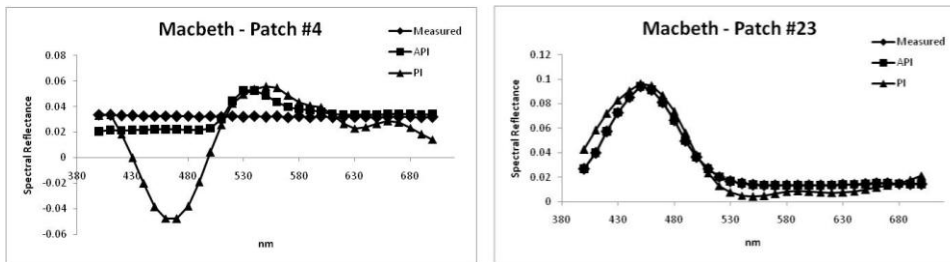
A comparison have been presented in Figure 4 showing the colorimetric error between Munsell [18], BabelColor [17] and Estimated API spectral Data for Macbeth ColorChecker CC which are less than 1 and within acceptable error range [23].

		API		PI	
<b>Kodak Chart (18 patches)</b>	<b>RMSE</b>	<b>Mean</b>	3.60592E-16	0.003603183	
		<b>STD</b>	5.56468E-16	0.001756709	
		<b>Max</b>	2.35017E-15	0.007424878	
	<b>ΔE<sub>00</sub> under D65</b>	<b>Mean</b>	1.42027E-14	0.128468081	
		<b>STD</b>	1.54E-14	0.070764767	
		<b>Max</b>	6.27972E-14	0.238846758	
<b>Macbeth CC (24 patches)</b>	<b>RMSE</b>	<b>Mean</b>	0.000427617	0.006133283	
		<b>STD</b>	0.001984025	0.006493265	
		<b>Max</b>	0.009728257	0.033024449	
	<b>ΔE<sub>00</sub> under D65</b>	<b>Mean</b>	0.008956815	0.163237335	
		<b>STD</b>	0.038608483	0.192726688	
		<b>Max</b>	0.188428881	0.976090041	
<b>Japanese Palette (173 patches)</b>	<b>RMSE</b>	<b>Mean</b>	0.001460306	0.004998975	
		<b>STD</b>	0.003768597	0.00461258	
		<b>Max</b>	0.028263637	0.032854126	
	<b>ΔE<sub>00</sub> under D65</b>	<b>Mean</b>	0.071885883	0.128116604	
		<b>STD</b>	0.236286633	0.123883428	
		<b>Max</b>	2.248539644	0.680269435	

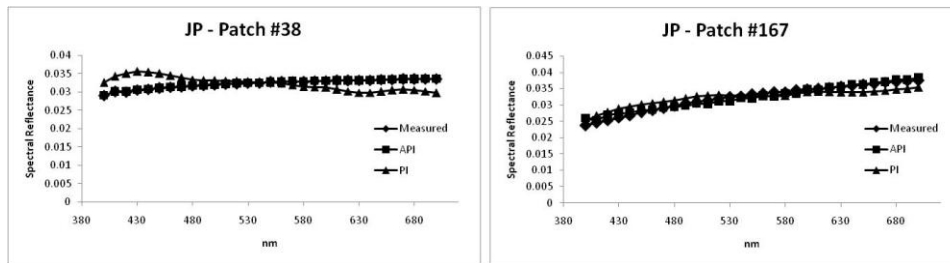
**TABLE 1:** Spectral and Colorimetric Errors for PI and API Methods.



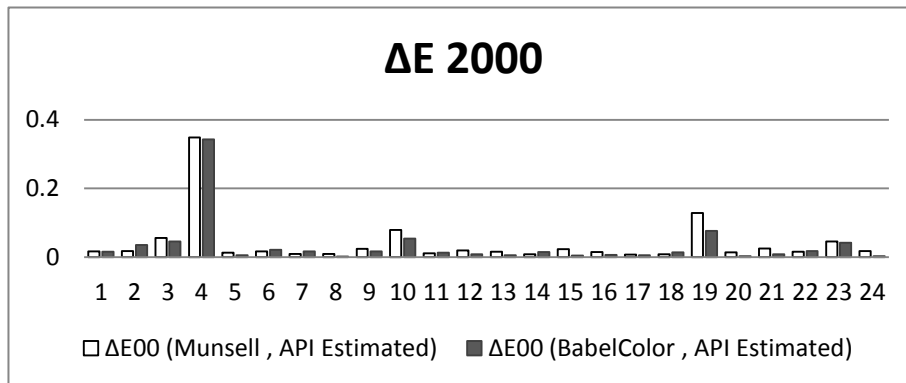
**FIGURE 3a:** Kodak Chart, Measured and reconstructed spectral reflectance of the API and PI methods (left) best case with spectral error =  $2.34E-17$  and colorimetric error =  $6.37E-17$ , (right) worst case with spectral error =  $2.35E-15$  and colorimetric error =  $6.28E-14$ .



**FIGURE 3b:** Macbeth Chart, Measured and reconstructed spectral reflectance of the API and PI methods (left) best case with spectral error =  $2.22E-17$  and colorimetric error =  $1.59E-17$ , (right) worst case with spectral error =  $0.09728$  and colorimetric error =  $0.188429$ .



**FIGURE 3c:** Japanese Palette, Measured and reconstructed spectral reflectance of the API and PI methods (left) best case with spectral error =  $1.42E-05$  and colorimetric error =  $0.000713$ , (right) worst case with spectral error =  $0.028264$  and colorimetric error =  $2.24854$ .



**FIGURE 4:** Comparison between Munsell, BabelColor and Estimated API spectral Data for Macbeth ColorChecker CC target (24 Patches) using Delta E2000 – colorimetric error.

#### 4. DISCUSSION

Since the acquisition of good quality spectral information is one of the main concerns of color systems, exhaustive research has been conducted for the acquisition, processing and estimation and even evaluation of spectral information. Spectral estimation methods as API depend on the accuracy of the spectral data in the reference database which depends on the spectral properties of the system calibration target [19].

Once the spectral information is acquired either measured by spectrometers, published by standard color targets manufacturers or estimated by spectral estimation methods as in this research, due attention has to be paid for the evaluation of the spectra. Delta E2000 [11:13,20] has been used for the evaluation and comparison of different estimation methods as it is a quantitative evaluation of color differences in LAB color space which matches with the future aims of this research in the reconstruction of color [21].

The performance of the proposed method (Adaptive Pseudo-Inverse) was tested by three different datasets and the results of the spectral estimation have been compared for the original and proposed methods using spectral and colorimetric errors (Table 1), Results showed noticeable improvement of the estimation accuracy. Moreover, the resulted spectra have been compared with other different Spectral data for GretagMacbeth Colorchecker CC according to the colorimetric error (Figure 4) which showed a potential improvement in the quality of the color reconstruction.

#### 5. CONCLUSION AND FURTHER WORK

This research introduced integrated approach for the reconstruction of spectral reflectance by the combination between the original Pseudo-Inverse method and the adaptive selection of samples as stated in Adaptive Wiener introduced by Xin and Shen. By the adaptive selection of training samples according to their spectral similarity to the verification sample, a new transformation matrix has been calculated for the estimation of spectral reflectance which improved the accuracy of the spectral estimation.

Further work would be developed for the comparison and evaluation of API method with other spectral estimation methods such as wiener, adaptive wiener...etc. Also, it is worth mentioning that more investigation about the appropriate L value (number of selected training samples) should be performed to achieve least spectral and colorimetric errors which consequently lead to better accuracy to be used in further research in the development of a portable multi-spectral system for the pigment identification and color reproduction [22].

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