A DUAL TREE COMPLEX WAVELET TRANSFORM CONSTRUCTION AND ITS APPLICATION TO IMAGE DENOISING

Sathesh sathesh_ece@yahoo.com

Assistant professor / ECE / School of Electrical Science
Karunya University, Coimbatore, 641114, India

Samuel Manoharan samuel1530@gmail.com

Phd Scholar / ECE / School of Electrical Science
Karunya University, Coimbatore, 641114, India

Abstract

This paper discusses the application of complex discrete wavelet transform (CDWT) which has significant advantages over real wavelet transform for certain signal processing problems. CDWT is a form of discrete wavelet transform, which generates complex coefficients by using a dual tree of wavelet filters to obtain their real and imaginary parts. The paper is divided into three sections. The first section deals with the disadvantage of Discrete Wavelet Transform (DWT) and method to overcome it. The second section of the paper is devoted to the theoretical analysis of complex wavelet transform and the last section deals with its verification using the simulated images.

Keywords: Complex Discrete Wavelet Transform (CDWT), Dual-Tree, Filter Bank, Shift Invariance, Optimal Thresholding.

1. INTRODUCTION

The application of wavelets to signal and image compression and to denoising is well researched. Orthogonal wavelet decompositions, based on separable, multirate filtering systems have been widely used in image and signal processing, largely for data compression. Kingsbury introduced a very elegant computational structure, the dual - tree complex wavelet transform [5], which displays near-shift invariant properties. Other constructions can be found such as in [11] and [9]. As pointed out by Kingsbury [5], one of the problems of mallat-type algorithms is the lack of shift invariance in such decompositions. A manifestation of this is that coefficient power may dramatically re –distribute itself throughout subbands when the input signal is shifted in time or in space.

Complex wavelets have not been used widely in image processing due to the difficulty in designing complex filters which satisfy a perfect reconstruction property. To overcome this, Kingsbury proposed a dual-tree implementation of the CWT (DT CWT) [7], which uses two trees of real filters to generate the real and imaginary parts of the wavelet coefficients separately. The two trees are shown in Fig. 3 for 1D signal. Even though the outputs of each tree are downsampled by summing the outputs of the two trees during reconstruction, the aliased components of the signal can be suppressed and approximate shift invariance can be achieved. In this paper CDWT, which is an alternative to the basic DWT the outputs of each tree are downsampled by summing the outputs of the two trees during reconstruction and the aliased components of the signal are suppressed and approximate shift invariance is achieved. The DWT suffers from the following two problems.

- Lack of shift invariance - this results from the down sampling operation at each level. When the input signal is shifted slightly, the amplitude of the wavelet coefficients varies so much.
- Lack of directional selectivity - as the DWT filters are real and separable the DWT cannot distinguish between the opposing diagonal directions.

These problems hinder the use of wavelets in other areas of image processing. The first problem can be avoided if the filter outputs from each level are not down sampled but this increases the computational costs significantly and the resulting undecimated wavelet transform still cannot distinguish between opposing diagonals since the transform is still separable. To distinguish opposing diagonals with separable filters the filter frequency responses are required to be asymmetric for positive and negative frequencies. A good way to achieve this is to use complex wavelet filters which can be made to suppress negative frequency components. The CDWT has improved shift-invariance and directional selectivity than the separable DWT.

The work described here contains several points of departure in both the construction and application of dual tree complex wavelet transform to feature detection and denoising.

2. DESIGN OVERVIEW

The dual-tree CWT comprises of two parallel wavelet filter bank trees that contain carefully designed filters of different delays that minimize the aliasing effects due to downsampling[5]. The dual-tree CDWT of a signal x(n) is implemented using two critically-sampled DWTs in parallel on the same data, as shown in Fig. 3. The transform is two times expansive because for an N-point signal it gives 2N DWT coefficients. If the filters in the upper and lower DWTs are the same, then no advantage is gained. So the filters are designed in a specific way such that the subband signals of the upper DWT can be interpreted as the real part of a complex wavelet transform and subband signals of the lower DWT can be interpreted as the imaginary part. When designed in this way the DT CDWT is nearly shift invariant, in contrast to the classic DWT.

3. TRANSLATION INVARIANCE BY PARALLEL FILTER BANKS

The orthogonal [8] two-channel filter banks with analysis low-pass filter given by the *z*-transform $H_0(z)$, analysis highpass filter $H_1(z)$ and with synthesis filters $G_0(z)$ and $G_1(z)$ is shown in figure.1

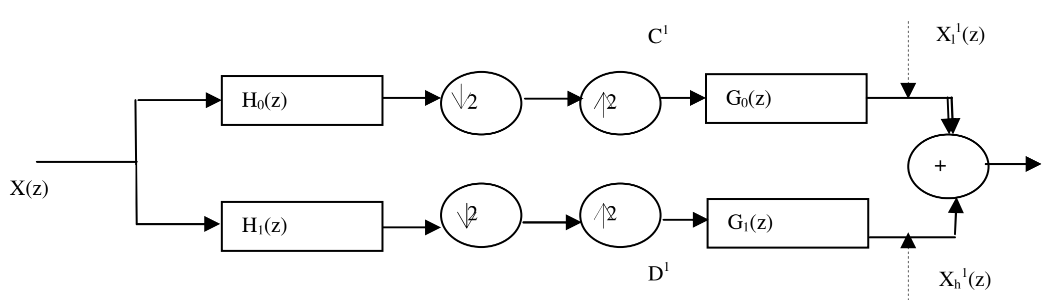

Figure 1: DWT Filter Bank

For an input signal X(z), the analysis part of the filter bank followed by upsampling produces the low-pass and the high-pass coefficients respectively, and decomposes the input signal into a low frequency part $X_l^1(z)$ and a high frequency part $X_h^1(z)$, the output signal is the sum of these two components.

$$C^1(z^2) = \left(\frac{1}{2}\right)\left(X(z)H_0(z) + X(-z)H_0(-z)\right) \quad (1)$$

$$D^1(z^2) = \left(\frac{1}{2}\right)\left(X(z)H_1(z) + X(-z)H_1(-z)\right) \quad (2)$$

$$Y(z) = X_l^1(z) + X_h^1(z) \quad (3)$$

Where

$$X_l^1(z) = C^1(z^2)G_0(z)$$

$$= \left(\frac{1}{2}\right)\left(X(z)H_0(z)G_0(z) + X(-z)H_0(-z)G_0(z)\right) \quad (4)$$

$$X_h^1(z) = D^1(z^2)G_1(z)$$

$$= \left(\frac{1}{2}\right)\left(X(z)H_1(z)G_1(z) + X(-z)H_1(-z)G_1(z)\right) \quad (5)$$

This decomposition is not shift invariant due to the terms in X(−z) of eqn 4 and eqn 5, respectively, which are introduced by the downsampling operators. If the input signal is shifted, for example $z^{-1}X(z)$, the application of the filter bank results in the decomposition

$$z^{-1}X(z) = \tilde{X}_l^1(z) + \tilde{X}_h^1(z) \quad (6)$$

For an input signal $z^{-1}X(z)$ we have

$$C^1(z^2) = \left(\frac{1}{2}\right)\left(z^{-1}X(z)H_0(z) + (-z^{-1})X(-z)H_0(-z)\right) \quad (7)$$

and

$$\tilde{X}_l^1(z) = \left(\frac{1}{2}\right)\left(X(z)H_0(z)G_0(z) - X(-z)H_0(-z)G_0(z)\right) \quad (8)$$

and similarly for the high-pass part, which of course is not the same as $z^{-1}X_l^1(z)$ if we substitute for z^{-1} in eqn 4. From this calculation it can be seen that the shift dependence is caused by the terms containing X(−z), the aliasing terms.

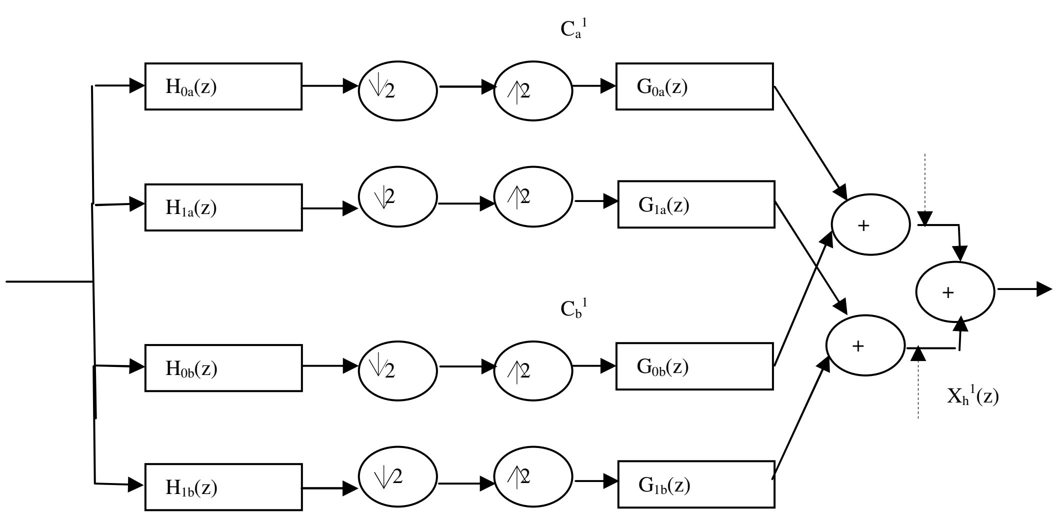

Figure 2: One level complex dual tree.

One possibility to obtain a shift invariant decomposition can be achieved by the addition of a filter bank to figure 1 with shifted analysis filters $z^{-1}H_0(z)$, $z^{-1}H_1(z)$ and synthesis filters $zG_0(z)$, $zG_1(z)$ and subsequently taking the average of the lowpass and the highpass branches of both filter banks as shown in figure 2.

If we denote the first filter bank by index a and the second one by index b then this procedure implies the following decomposition

$$X(z) = X_l^1(z) + X_h^1(z) \tag{9}$$

where for the lowpass channels of tree a and tree b we have

$$X_l^1(z) = \left(\frac{1}{2}\right)\left(C_a^1(z^2)G_{0a}(z) + C_b^1(z^2)G_{0b}(z)\right)$$
$$= \left(\frac{1}{2}\right) X(z)\, H_0(z) G_0(z) \tag{10}$$

and similarly for the high-pass part. The aliasing term containing X(-z) in X_l^1 has vanished and the decomposition becomes indeed shift invariant.

Using the same principle for the design of shift invariant filter decomposition, Kingsbury suggested in [4] to apply a 'dual-tree' of two parallel filter banks are constructed and their bandpass outputs are combined. The structure of a resulting analysis filter bank is shown in Fig. 3, where index *a* stands for the original filter bank and the index *b* is for the additional one. The dual-tree complex DWT of a signal x(n) is implemented using two critically-sampled DWTs in parallel on the same data.

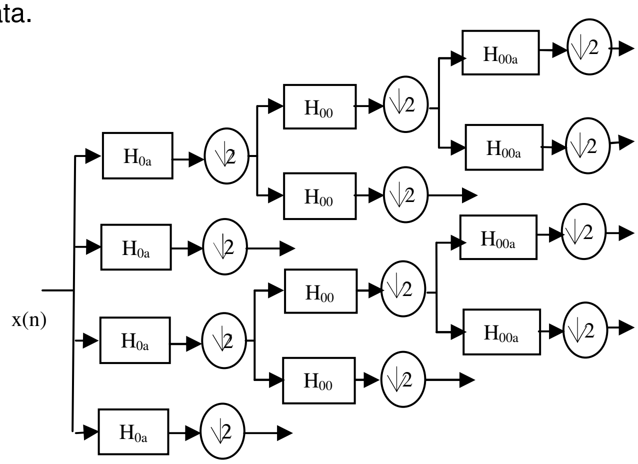

Figure 3: Three level Complex dual tree

In one dimension, the so-called dual-tree complex wavelet transform provides a representation of a signal x(n) in terms of complex wavelets, composed of real and imaginary parts which are in turn wavelets themselves. In fact, these real and imaginary parts essentially form a quadrature pair.

H_{0a}	H_{1a}	H_{0b}	H_{1b}
0	0	0.01122679	0
-0.08838834	-0.01122679	0.01122679	0
0.08838834	0.01122679	-0.08838834	-0.08838834
0.69587998	0.08838834	0.08838834	-0.08838834
0.69587998	0.08838834	0.69587998	0.69587998
0.08838834	-0.69587998	0.69587998	-0.69587998
-0.08838834	0.69587998	0.08838834	0.08838834
0.01122679	-0.08838834	-0.08838834	0.08838834
0.01122679	-0.08838834	0	0.01122679
0	0	0	-0.01122679

TABLE 1: First Level DWT Coefficients

The dual-tree CDWT uses length-10 filters [6], the table of coefficients of the analyzing filters in the first stage is shown in table 1 and the remaining levels are shown in table 2. The reconstruction filters are obtained by simply reversing the alternate coefficients of the analysis filters.

To extend the transform to higher-dimensional signals, a filter bank is usually applied separably in all dimensions. To compute the 2D CWT of images these two trees are applied to the rows and then the columns of the image as in the basic DWT.

Tree a		Tree b	
H_{00a}	H_{01a}	H_{00b}	H_{01b}
0.03516384	0	0	-0.03516384
0	0	0	0
-0.08832942	-0.11430184	-0.11430184	0.08832942
0.23389032	0	0	0.23389032
0.76027237	0.58751830	0.58751830	-0.76027237
0.58751830	-0.76027237	0.76027237	0.58751830
0	0.23389032	0.23389032	0
-0.11430184	0.08832942	-0.08832942	-0.11430184
0	0	0	0
0	-0.03516384	0.03516384	0

TABLE 2: Remaining Levels DWT Coefficients

This operation results in six complex high-pass subbands at each level and two complex low-pass subbands on which subsequent stages iterate in contrast to three real high-pass and one real low-pass subband for the real 2D transform. This shows that the complex transform has a coefficient redundancy of 4:1 or 2m : 1 in m dimensions. In case of real 2D filter banks the three highpass filters have orientations of 0^{θ}, 45^{θ} and 90^{θ}, for the complex filters the six subband filters are oriented at angles $\pm 15^{\theta}, \pm 45^{\theta}, \pm 75^{\theta}$. This is shown in figure 4.

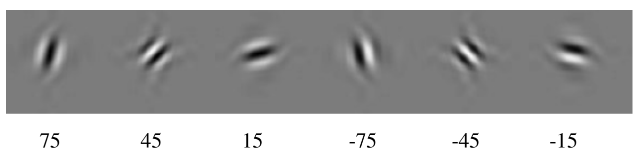

Figure 4: Complex filter response showing the orientations of the complex wavelets

The CDWT decomposes an image into a pyramid of complex subimages, with each level containing six oriented subimages resulting from evenly spaced directional filtering and subsampling, such directional filters are not obtainable by a separable DWT using a real filter pair but complex coefficients makes this selectivity possible.

4. RESULTS AND DISCUSSION

The shift invariance and directionality of the CWT may be applied in many areas of image processing like denoising, feature extraction, object segmentation and image classification. Here we shall consider the denoising example. For denoising a soft thresholding method is used. The choice of threshold limits σ for each decomposition level and modification of the coefficients is defined in the following equation.

$$\bar{c}_s(k) = \{\sin c(k)(|c(k)| - \sigma) \text{ if } |c(k)| > \sigma$$
$$\bar{c}_s(k) = 0 \;\; if \;\; |c(k)| \leq \sigma$$
(11)

To compare the efficiency of the DWT with the basic DWT the quantitative mean square error (MSE) is used. In all cases the optimal thresholds points σ were selected to give the minimum square error from the original image, showing a great effectiveness in removing the noise compared to the classical DWT as shown in table 3.

Figure 5: (a) Input Image, (b) Denoised with real CWT,(c) Denoised with dual tree CWT

From figure 5(b) it may be seen that DWT introduces prominent worse artifacts, while the DT CWT provides a qualitatively restoration with a better optimal minimum MSE error.

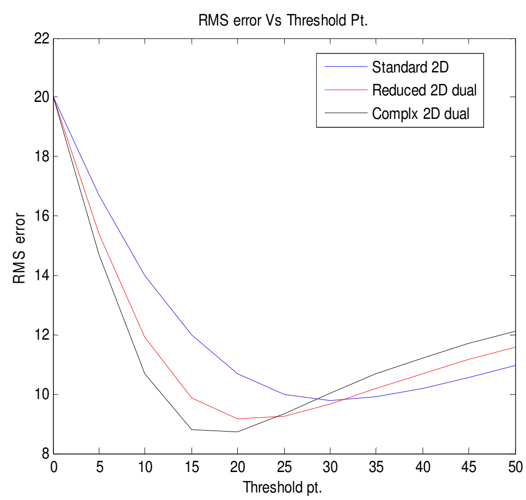

Figure 6: Optimal threshold points for the three different methods

The table 3 gives the comparison between the various methods in terms of their Mean Square Error (MSE) and Signal-to-Noise Ratio (SNR) Values.

Type of method	MSE	SNR [dB]
noisy image	0.0418	20.8347
DWT	0.0262	25.4986

real CWT	0.0255	25.7601
CWT	0.0240	26.3751

TABLE 3: Mean Square Error (MSE) and Signal – to – Noise Ratio (SNR) Values

The DT CWT is shift invariant and forms directionally selective diagonal filters. These properties are important for many applications in image processing including denoising, deblurring, segmentation and classification. In this paper we have illustrated the example of the application of complex wavelets for the denoising of Lena images. To obtain further improvements, it is also necessary to develop principled statistical models for the behavior of features under addition of noise, and their relationship to the uncorrupted wavelet coefficients. This remains to be done.

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