An Edge Detection Method for Hexagonal Images

Kazi Mostafa *kazimostafa@gmail.com*
Mechanical Engineering
Shandong University of Technology
Zibo, China

Innchyn Her *her@mail.nsysu.edu.tw*
Mechanical & Electro-mechanical Engineering
National Sun Yat-sen University
Kaohsiung, 804, Taiwan

Abstract

This paper presents a morphological image processing operation for hexagonally sampled images and proposes a new edge detection method for these images by using a grayscale morphology. This is achieved by applying morphological gradient operators and multiscale top-hat transformations (white and black top-hat transformations) to hexagonal images. The proposed study includes a method for converting hexagonally sampled images as well as the processing and subsequent display of images on a hexagonal grid. Performance evaluation were performed to assess the proposed method. The proposed study shows that a method of edge enhancement by applying three by three hexagonal structuring element achieves results superior to those of a rectangular images. The results indicated that the proposed edge detection algorithms improved substantially after implementation of the edge enhancement method.

Keywords: Hexagonal Grid, Mathematical Morphology, Top-hat Transform, Edge Detection.

1. INTRODUCTION

Edge detection and enhancement [1, 2] are essential procedures in the fields of image processing, computer vision, robotics, and automation. Several techniques are used for edge detection. Amongst these techniques, morphological image processing is a conventional approach. However, the success of edge detection relies heavily on the shape of the object in an image. Morphological gradient often performs poorly when applied to rectangular image curved objects. To overcome this limitation, high-resolution grids are frequently used; however, using high-resolution images is expensive and reduces computational performance. Sampling on a hexagonal lattice is an alternative solution for overcoming this difficulty [3, 4]. Hexagonal image processing provides decreased storage and computation time, increased coding efficiency, less quantisation error, the property of equidistance, and consistent connectivity [5]. In addition, the sampling efficiency of a hexagonal lattice exceeds that of a rectangular lattice [6], and hexagonal images have been determined to be more aesthetically pleasing to the human eye. Recently, many companies have introduced image acquisition and display devices which use increased numbers of rectangular pixels per inch. These devices require additional computational power and storage capacity. Some applications, such as real-time monitoring programmes, require the use of low-resolution imaging to save computational power and storage. Hexagonal grid systems are an effective alternative for solving these problems. The main obstacle limiting the use of hexagonal images has been the lack of hardware for capturing and displaying hexagonal-grid-based images [5]. To overcome this obstacle, a resampling method [6-9] has been developed to convert rectangular grid images to hexagonal counterparts. This method is based on image resampling to generate a hexagonally sampled image from a traditional rectangular grid image.

A detailed overview of morphological operations such as dilation, erosion, opening, and closing is provided in [10]. In addition, a morphological edge detection algorithm was proposed therein to detect the edges of lung computed tomography images with salt-and-pepper noise [11]. The method is based on a combination of dilation, erosion, opening, and closing; dilation and erosion are employed to detect edges, and opening-closing operations are used as preprocessing procedures to eliminate noise. In recent studies, morphological edge detection has been proposed and compared with Prewitt, Laplacian of Gaussian, and Canny edge operators [12, 13]. Moreover, multiscale top-hat transformations [14, 15] based on structuring elements (SEs) have been constructed for various applications.

Hexagonal image processing is as old as image processing itself but sufficient research has been lacking because the availability of hexagonal pixel display devices is limited. In this paper, an edge detection method that involves applying a grayscale morphology to hexagonally sampled images is proposed to alleviate this problem. The proposed edge detection and enhancement method entails converting rectangular grid images into hexagonal grid image and is suitable and convenient for traditional display devices.

This guideline is used for all journals. These are the manuscript preparation guidelines used as a standard template for all journal submissions. Author must follow these instructions while preparing/modifying these guidelines. This guideline is used for all journals. This guideline is used for all journals. These are the manuscript preparation guidelines used as a standard template for all journal submissions. Author must follow these instructions while preparing/modifying these guidelines. This guideline is used for all journals.

2. RESEARCH OBJECTIVES

This paper proposes a design framework to overcome the limitations of edge detection by using rectangular grid imaging. In our framework, a morphological gradient operator and multiscale top-hat transformations to hexagonally sampled images are applied. To exploit the advantages of hexagonal images for broader applications, this study developed an efficient method for detecting edges by using a grayscale morphological operator. Our objectives are summarized as follows:

- To provide a comprehensive guideline of hexagonally sampled images converted from rectangular images, and the processing and display of images on hexagonal grids. Image processing involves applying grayscale morphological operations to hexagonal counterparts to detect edges; specifically, morphological gradient operators are to detect edges and multiscale top-hat transformations are employed to enhance edge detection performance.
- To develop multiscale SEs with various sizes for hexagonally sampled images and then select the better structuring element for use in the proposed method.
- To evaluate the performance of the proposed algorithm by using qualitative comparison and the linear index of fuzziness.

3. BACKGROUND

3.1 Morphological Image Processing

Mathematical morphology is an approach for extracting geometrical features from signals based on set theory [10]. Morphological edge detection [16] has generally been performed using SEs [17, 18]. The most basic morphological operations are dilation and erosion. Grayscale erosion and dilation produce results identical to those of nonlinear minimum and maximum filters. The general effects of performing dilation on a grayscale image are that the output image tends to be brighter than the input, and dark details are reduced. The dilation and erosion of grayscale image A by SE B is shown as follows:

$$(A \oplus B)(s,t) = \max\{ f(s-x, t-y) + B(x,y) \mid (s-x),(t-y) \in D_A; (x,y) \in \mathrm{D}_B \}, \tag{1}$$

$$(A \Theta B)(s,t) = \min\{ f(s+x, t+y) - B(x,y) \mid (s+x),(t+y) \in D_A; (x,y) \in \mathrm{D}_B \},$$

3.2 Hexagonal Image Processing

A rectangular grid of sampling points and pixels is shown in Figure 1(a). Several types of sampling scheme were listed by Whitehouse 19. Figures 1(b, c) show hexagonally sampled image, which are characterized by the hexagonal tiling. Hexagonal grids have numerous advantages in many applications, e.g., robot vision [20], deep-space imaging [21], image processing. For a morphological operator, a 3 by 3 SE is the closest fit for a circular SE in a rectangular grid image. However, the use of such an element to erode curved objects can introduce unwanted discontinuities. This is because the elements within the SE are at different distances from the center. The consistent connectivity of pixels in hexagonal images is an attractive feature which improves circular SE definition, as shown in Figure 2. Moreover, computation of a neighborhood is simple in a hexagonal lattice, as shown in Figure 3.

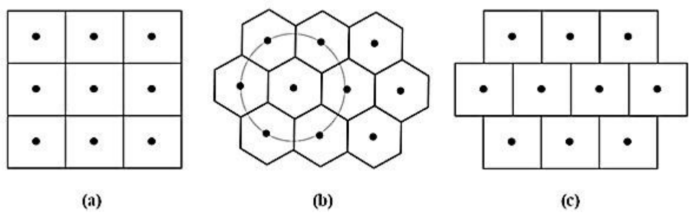


FIGURE 1: Image tiling: (a) square grid; (b) hexagonal grid with hexagonal tiles; (c) hexagonal grid with rectangular tiles.

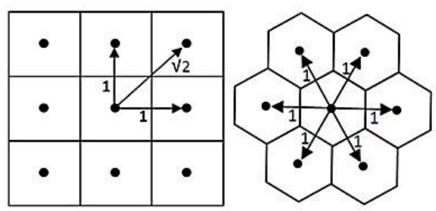


FIGURE 2: Distance in a square grid & hexagonal grid.

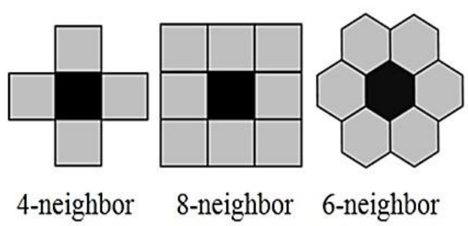


FIGURE 3: Neighborhood relationships.

4. METHODOLOGY

The current study was divided into three phases as depicted in Figure 4. We first used an image resampling method to convert the grid of an image from rectangular to hexagonal one and then performed edge detection by using the morphological gradient method. Edges were enhanced by applying multiscale top-hat transformations. Finally, performance evaluation algorithms were employed to assess the results of the proposed methods. We converted rectangular grid image to hexagonal grid by using simulated hexagonal grid method. Simulated hexagonal grid can be created through clusters of rectangular sub-pixels. Many rectangular grids can be combined together to create clusters of sub-pixels (as shown in Figure 5). For example, d = 7; is composed of 120 rectangular grids to create one hexagonal pixel. A cluster of sub-pixels will closely resemble the shape of hexagon. D7 simulated hexagonal conversion method performed better than others. The geometric properties of D7 hexagonal pixel are similar to hexagon shape and it showed equivalent inter-pixel distance and smooth curved edges. Moreover, rectangular grid may perform well only for horizontal and vertical direction. Proposed D7 hexagonal grid performed not

only for horizontal and vertical lines but also oblique edges. The research methodology can be summarized as follows:
Step 01: Image resampling is conducted to convert rectangular grid input images into hexagonally sampled images.
Step 02: Several SEs of various sizes are used in dilation and erosion.
Step 03: Edges are detected by calculating the differences between the dilated and eroded images of all SEs and then choosing the better SE.
Step 04: Detected edges are enhanced by applying multiscale top-hat transformations.
Step 05: Performance evaluation is conducted using QC and the index of fuzziness.

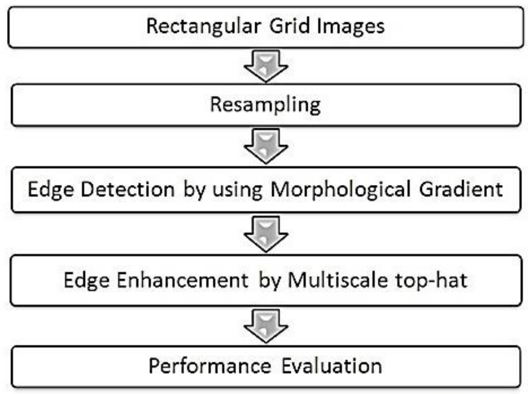


FIGURE 4: System diagram for performing image processing on a hexagonally sampled grid.

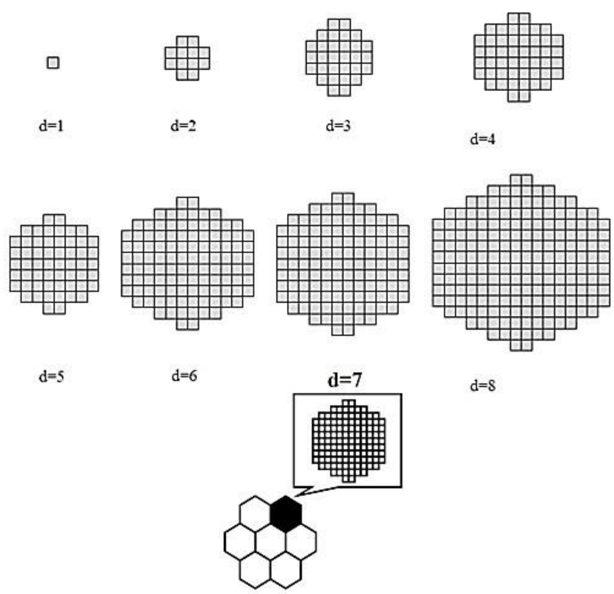


FIGURE 5: Simulated hexagonal grid.

4.1 Resampling (Square to Hexagonal)

In this study, we employed a simulated hexagonal grid to convert the sampling of images from rectangular to hexagonal. A simulated hexagonal grid can be created using clusters of rectangular subpixels5, which are created by combining several rectangular grids, as shown in Figure 5. For example, the D7 method entails combining 120 rectangular grids to create one hexagonal pixel. In this resampling method, a cluster of subpixels closely resembles a hexagon. Each simulated hexagonal pixel is formed by 120 sub-pixels arranged. The light intensity of each constructed hexagonal pixel can be easily computed as the average of the intensities of the 120 sub-pixels forming the hexagonal pixel. Hence, the number of hexagonal pixels is less than the number of square pixels to cover the same image. From the observation result obtained, it is claimed that fewer sampling points (or pixels) are required with a hexagonal structure to maintain equal amount of image information (or the same image resolution) with the traditional square structure. The hexagonal pixels constructed in the way above will not lose image resolution. Following the resampling procedure, of which the result is shown in Figure 6(b), the dimensions were 512 × 512. A small portion of the enlarged view was added at the right side of the standard test image Lena to highlight the rectangular and hexagonal grids. To increase the computation speed, we converted these dimensions to 256 × 256. We applied sub pixel-size and D7 method to generate hexagonal pixels to obtain the corresponding intensity values of a simulated hexagonal grid.

FIGURE 6: Image resampling (a) rectangular grid image (b) hexagonal grid image (512 by 512).

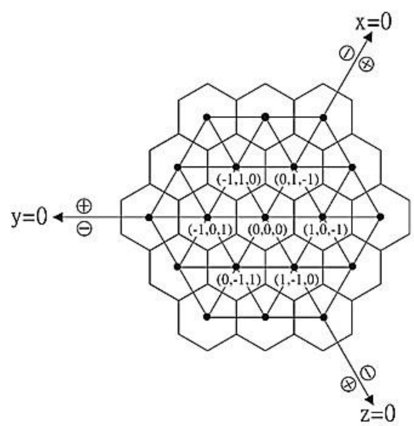


FIGURE 7: Symmetrical hexagonal coordinate frame.

However, hexagonal pixels cannot be labelled in a normal column-row order as in a rectangular grid. To properly address and store the hexagonal image data, we applied symmetrical coordinate systems 7. A three-coordinate system was developed to represent hexagonal data. As shown in Figure 7, the system uses a three-tuple to represent the displacement from each of the major axes of the hexagon. The three coordinates are x = 0, y = 0, and z = 0.

4.2 Hexagonal Structuring Elements

After image conversion, the application of morphological operations to hexagonal images to detect edges was conducted. The performance of the morphological operation depends heavily on the size and shape of the SE. When certain SEs are used, erosion and dilation can be achieved more rapidly and with more favourable results. In this paper, four (3 × 3, 5 × 5, 7 × 7 and 9 × 9) new hexagonal grid SEs are proposed to detect and enhance edges in hexagonally sampled images, as shown in Figure 8.

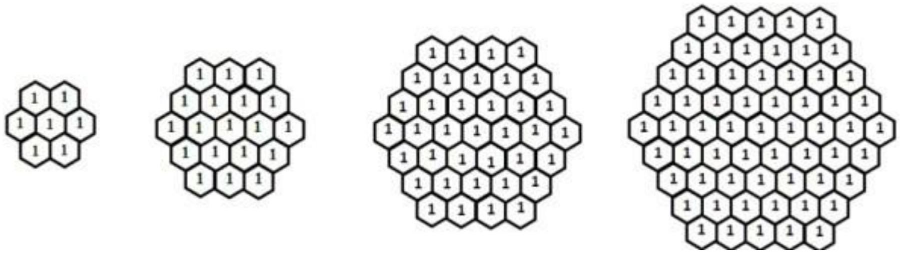


FIGURE 8: Proposed structuring elements.

4.3 Hexagonal Grayscale Morphological Operator

The most basic morphological image processing operations are dilation and erosion. Grayscale erosion and dilation produce results identical to those of nonlinear minimum and maximum filters. This study presents grayscale dilation and erosion on hexagonally sampled images, as shown in Figure 9. Grayscale dilation and erosion were achieved by applying four new hexagonal SEs (3 × 3, 5 × 5, 7 × 7, 9 × 9) to dilate and erode standard test images (Lena, Barbara, Pepper). Figures 10 and 11 show 3 × 3 and 5 × 5 hexagonal SE operations applied to the Lena test image. After dilation and erosion, closing and opening, which are sequential combinations of dilation and erosion, were performed by using aforementioned hexagonal SEs.

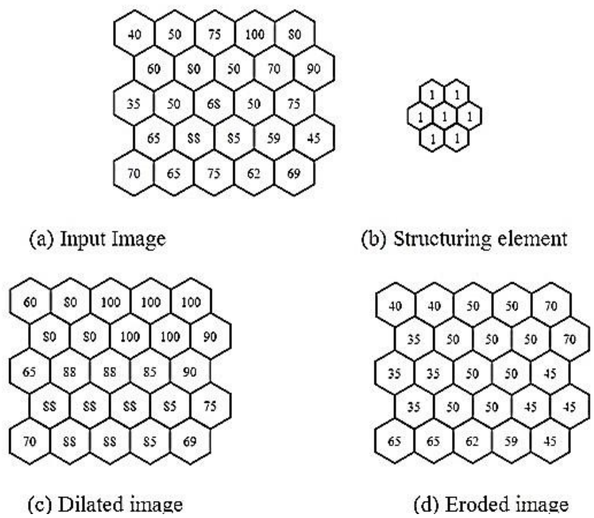


FIGURE 9: Dilation and Erosion on hexagonal image.

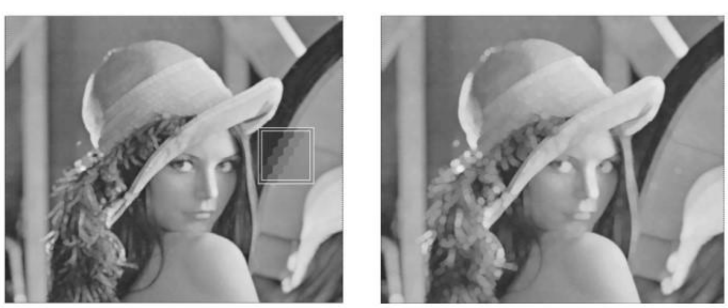


FIGURE 10: Dilated image (a) 3x3 SE, (b) 5x5 SE.

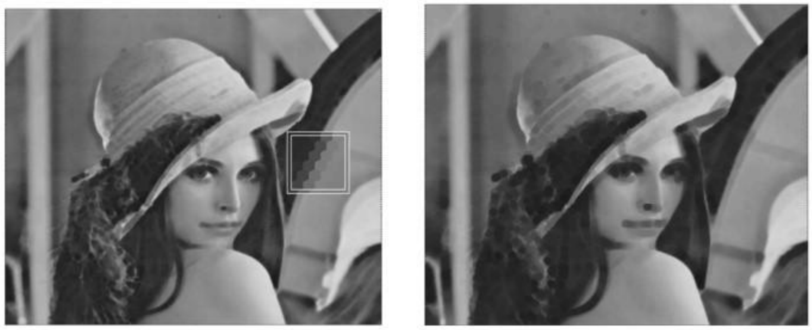


FIGURE 11: Eroded Image (a) 3x3 SE (b) 5x5 SE.

Conventional mathematical morphology entails implementing single-scale analysis. A fixed SE is used based on prior knowledge. However, prior knowledge is not always available for some applications. Thus, multiscale morphological analysis [22] seems to be more favourable than single-scale analysis. Multiscale morphology can be implemented by using hexagonal SEs with various scales. Let n (n= 1, 2, …. nmax) be the scale and B be the unit SE. Thus, the SE used at scale n can be defined as:

$$nB = B\underbrace{\oplus B \oplus B \oplus B \oplus \cdots \oplus B}_{n-1 times}, \qquad (2)$$

where B is the SE and n is the number of operations.

Dilation and erosion can then be obtained:
Multiscale Dilation = A⊕nB, and Multiscale Erosion = AΘnB,
where A is the input image, B is the SE, and n is the number of operations.

Correspondingly, opening and closing are considered extensions of basic dilation and erosion operations. Opening consists of erosion followed by dilation. Closing is dilation followed by erosion. Multiscale opening and closing can be denoted by:

Multiscale Opening = AonB= (AΘnB)⊕nB, (3)
Multiscale Closing = A●nB= (A⊕nB)ΘnB,

4.4 Edge Detection (Morphological Gradient)
A morphological gradient is the difference between the dilation and the erosion of a given image. The most basic morphological gradient operator is shown as follows:

Edge = (A⊕B) – (AΘB), (4)

A few more operators based on the most basic operator are available; for example,

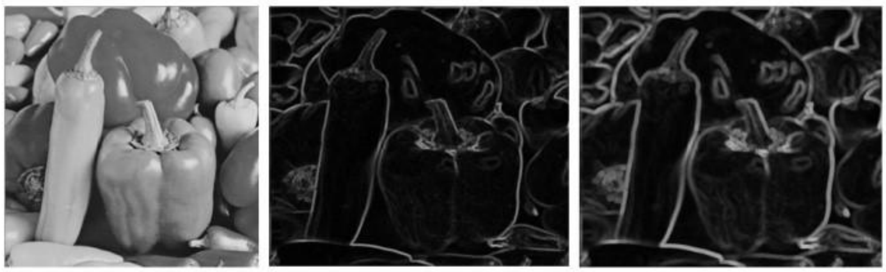
Edge = (A●B) – (AoB), (5)

Multiscale morphological gradient operators at scale n can be defined as the differences between dilation and erosion or between closing and opening:

Multiscale Gradient = (A⊕nB) – (AΘnB), (6)
Multiscale Gradient = (A●nB) – (AonB),

After dilation, erosion, opening, and closing, the proposed method involves detecting edges by applying a morphological gradient operator to a hexagonal image. The Lena test image was dilated and eroded by applying 3 × 3, 5 × 5, 7 × 7, and 9 × 9 hexagonal SEs. The eroded image was then subtracted from the dilated image. The transformed Lena, Barbara, and Pepper images exhibited numerous edges, as shown in Figures 12, 13, 14, and 15.

FIGURE 12: Edge image by using hexagonal morphological gradient operator (Lena) (a) 3x3; (b) 5x5.

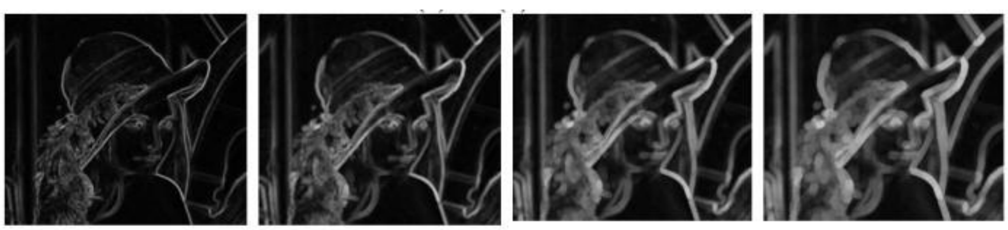


(a) (b)

FIGURE 13: Edge image by using hexagonal morphological gradient operator (Pepper) (a) 3x3; (b) 5x5.

(a) (b)

FIGURE 14: Edge image by using hexagonal morphological gradient operator (Barbara) (a) 3x3(b) 5x5.



(a) (b) (c) (d)

FIGURE 15: Multiscale hexagonal morphological gradient 3x3 SE (Lena) (a) n=1(b) n=2(c) n=3(d) n=4.

4.5 Edge Enhancement (Top-Hat Transformation)

To enhance the edges detected from single- and multiscale morphological gradient operations, a multiscale top-hat transformation algorithm is used. Recently, multiscale theory has become an attractive option for image processing [23, 24]. Based on opening and closing, the classical top-hat transformation defined [10] by White (WTH) entails computing the morphological opening of an image and then subtracting the result from the original image. The WTH transformation returns elements that are smaller than the SEs and brighter than their surroundings. The black top-hat (BTH) transformation involves subtracting the original image from morphological closing results. Thus, top-hat transformations using multiscale hexagonal SEs [14, 15] yield a better output. The WTH and BTH of gray image $f(x, y)$ by applying SE $B(u, v)$ are defined as follows:

$$\begin{aligned} WTH(x,y) &= f(x,y) - \min\big((f \ominus B) \oplus B, f(x,y)\big), \\ BTH(x,y) &= \max((f \oplus B) \ominus B, f(x,y)) - f(x,y), \end{aligned} \tag{7}$$

where $\ominus$ $\oplus$ are erosion and dilation operations, respectively, and B is a multiscale hexagonal SE with various sizes (3 × 3, 5 × 5, 7 × 7 and 9 × 9). According to the formula presented previously, light and dark image regions yield the highest gray values. Therefore, they can be defined as follows:

MW = max (WTH1, WTH2, … , WTHm), (8)

MB = max (BTH1, BTH2, … , BTHm),

where m is the size of hexagonal SEs (e.g., m1 is a 3 × 3 SE, m2 is a 5 × 5 SE, m3 is a 7 × 7 SE, and m4 is a 9 × 9 SE). Thus, edge can be enhanced as follows:

Enhanced Edges = (Original Image x W1) + (MW x W2) – (M B x W3), (9)

where W1, W2, W3 are the weights for adjusting the results for various applications. Using a low W1 and high W2 and W3 brightens the light regions. Thus, we set W1 = 1 and W2 and W3 = 3. The edge enhancement formula is applied to edges detected from a single-scale morphological gradient, as shown in Figures 12, 13, and 14. Enhanced edges are shown in Figures 16 and 17.

(a) (b)

FIGURE 16: Enhanced edges after applying top-hat transform; (a) 3x3(b) 5x5.

(a) (b)

FIGURE 17: Enhanced edges after applying top-hat transform (3x3 SE); (a) Pepper (b) Barbara.

5. PERFORMANCE EVALUATION

Two performance evaluation measures, namely the linear index of fuzziness [25] was applied to assess the results of the proposed method. The index of fuzziness was used to quantitatively assess the performance of the proposed algorithm. The linear index of fuzziness is based on spatial domain analysis and is defined as:

$$p_{xy} = \sin[\frac{\pi}{2} \text{ x } (1 - \frac{f_{xy}}{f_{max}})] \;; \quad \gamma = \frac{2}{MN}\sum_{x=1}^{m} \sum_{y=1}^{n} \min(p_{xy}, (1 - p_{xy})), \qquad (10)$$

where fxy and fmax represent the gray value of the pixel (x, y) and the maximum gray value of an image with size M × N, respectively. The fuzzy property is denoted by p_{xy}. A lower value of γ indicates a more desirable performance.

	SE	Lena	Pepper	Barbara
		Index of fuzziness	**Index of fuzziness**	**Index of fuzziness**
Hexagonal Grid	**3x3**	**0.064**	**0.062**	0.116
	5x5	**0.137**	**0.137**	0.203
	7x7	**0.201**	**0.205**	0.283
	9x9	**0.255**	**0.265**	0.353
Rectangular Grid	**3x3**	**0.098**	**0.091**	0.201
	5x5	**0.247**	**0.207**	0.302
	7x7	**0.301**	**0.306**	0.386
	9x9	0.359	0.369	0.464

TABLE 1: Comparison of different HSE edge detection using morphological gradient.

The linear index of fuzziness values for the morphological gradient edge detection operators of various SEs are listed in Table 1. Table 2 shows comparisons of the linear index of fuzziness values amongst the test images when the proposed edge enhancement method was used. A lower index of fuzziness value represents a more desirable result. Table 1 shows that 3 × 3 hexagonal SEs (HSEs) lower index of fuzziness. Thus, the 3 × 3 HSE was better than rectangular counterpart. We applied the proposed edge enhancement method to the four edge images listed in Table 1 showed in Table 2. For 3 × 3 HSE edges, the proposed method produced a lower index of fuzziness compared with rectangular grid image. Moreover, comparing Tables 1 and 2 revealed that edge results improved after the proposed edge enhancement method was applied. Conclusions were formulated based on the figures showing the enhanced edge images as well as the linear index of fuzziness values. Comparing the hexagonal SE (HSE) edge detection and enhanced edge method images clearly indicated that the proposed algorithm outperformed in edge detection and edge enhancement. The linear index of fuzziness demonstrate that the proposed 3 × 3 HSE method achieved the lowest index of fuzziness. A substantial difference was observed between the hexagonal and rectangular grids. As shown in Table 1 and 2, the index of fuzziness also indicated that the proposed method showed quantitatively favorable results. A comparison of the two grids revealed that, overall, the edge of the proposed hexagonal method was more continuous, well-defined and clearer.

	SE	Lena	Pepper	Barbara
		Index of fuzziness	Index of fuzziness	Index of fuzziness
Hexagonal Grid	**3x3**	**0.054**	**0.055**	0.101
	5x5	**0.126**	**0.105**	0.155
	7x7	**0.153**	**0.133**	0.189
	9x9	**0.181**	**0.175**	0.229
Rectangular Grid	**3x3**	**0.089**	**0.090**	0.177
	5x5	**0.168**	**0.110**	0.232
	7x7	**0.219**	**0.219**	0.286
	9x9	0.279	0.257	0.374

TABLE 2: Comparison of different HSE enhanced edge detection

6. CONCLUSION

Our aim was to explore hexagonal grid and compared with traditional rectangular grid. Moreover, to see the hexagonal grid would improve the performance of edge detection method for low resolution images. This paper presents a new edge detection and enhancement method based on hexagonally sampled images and a grayscale morphology. Edges were first detected by performing hexagonal morphological gradient operations. The edge enhancement method was subsequently applied to achieve better results. In both edge detection and enhancement, 3 × 3 hexagonal SE performed the most favorably. As shown in Tables 1 and 2, a three by three hexagonal structuring element achieves results superior to those of a rectangular image. Thus, hexagonal rather than rectangular images can be employed in edge detection by applying morphological operators to produce better results. The present study was designed to determine the effectiveness of a grayscale morphology in improving hexagonal images. The proposed method entails detecting edges by using a gray morphology and enhancing edges by using multiscale top-hat transformations. The results indicated that the edge detection performance improved significantly after the proposed edge enhancement method was implemented. In conclusion, from the performance evaluation and visual comparison, we determined that consistent and accurate gradient operators can be obtained by applying hexagonal lattices. The results clearly showed that the hexagonal grid performed adequately for curved objects as well as for horizontal and vertical lines.

7. REFERENCES

[1] Y. Liu and S. Zhou, A self-adaptive edge matching method based on mean shift and its application in video tracking. The Imaging Science Journal, 2014, 62(4), 206-216.

[2] Y. Xiong, J. Li, X. Zuo, and Z. Chen, Research on an Edge Detection Algorithm of Remote Sensing Image Based on Wavelet Enhancement and Morphology. Journal of Computers, 2014, 9(5), 1247-1252.

[3] S. Coleman, B. Scotney, and B. Gardiner: 'A Biologically Inspired Approach for Fast Image Processing', 2013, IAPR.

[4] S. A. Coleman, B. W. Scotney, and B. Gardiner: 'Design of Feature Extraction Operators for Use on Biologically Motivated Hexagonal Image Structures', MVA, 2009, 178-181.

[5] L. Middleton and J. Sivaswamy: 'Hexagonal image processing: A practical approach'; 2006, Springer.

[6] R. C. Staunton, The processing of hexagonally sampled images. Adv. Imaging Electron Phys, 2001, 119, 191-265.

[7] I. Her, A symmetrical coordinate frame on the hexagonal grid for computer graphics and vision. Journal of Mechanical Design, 1993, 115(3), 447-449.

[8] I. Her, Geometric transformations on the hexagonal grid. Image Processing, IEEE Transactions on, 1995, 4(9), 1213-1222.

[9] I. Her and C.-T. Yuan, Resampling on a pseudohexagonal grid. CVGIP: Graphical Models and Image Processing, 1994, 56(4), 336-347.

[10] J. Serra, Introduction to mathematical morphology. Computer vision, graphics, and image processing, 1986, 35(3), 283-305.

[11] Z. Yu-qian, G. Wei-hua, C. Zhen-cheng, T. Jing-tian, and L. Ling-Yun: 'Medical images edge detection based on mathematical morphology', Engineering in Medicine and Biology Society, 2005. IEEE-EMBS 2005. 27th Annual International Conference of the, 2006, IEEE, 6492-6495.

[12] B. Kaur and A. Garg: 'Mathematical morphological edge detection for remote sensing images', Electronics Computer Technology (ICECT), 2011 3rd International Conference on, 2011, IEEE, 324-327.

[13] T.-C. Su, M.-D. Yang, T.-C. Wu, and J.-Y. Lin, Morphological segmentation based on edge detection for sewer pipe defects on CCTV images. Expert Systems with Applications, 2011, 38(10), 13094-13114.

[14] X. Bai and F. Zhou, Analysis of different modified top-hat transformations based on structuring element construction. Signal Processing, 2010, 90(11), 2999-3003.

[15] X. Bai, F. Zhou, and B. Xue, Infrared image enhancement through contrast enhancement by using multiscale new top-hat transform. Infrared Physics & Technology, 2011, 54(2), 61-69.

[16] T. A. Mahmoud and S. Marshall, Edge-detected guided morphological filter for image sharpening. EURASIP Journal on Image and Video Processing, 2008, 2008.

[17] B. Chanda, M. K. Kundu, and Y. Vani Padmaja, A multi-scale morphologic edge detector. Pattern Recognition, 1998, 31(10), 1469-1478.

[18] J. GUO, S. PAN, and X.-j. HU, Edge Detection in Tobacco Leaf Image Based on Grayscale Morphology [J]. Computer Engineering, 2007, 21, 060.

[19] D. Whitehouse and M. Phillips, Sampling in a two-dimensional plane. Journal of Physics A: Mathematical and General, 1985, 18(13), 2465.

[20] R. C. Staunton: 'Hexagonal image sampling: A practical proposition', 1988 Robotics Conferences, 1989, International Society for Optics and Photonics, 23-27.

[21] J. Allen: 'Perfect reconstruction filter banks for the hexagon grid', Information, Communications and Signal Processing, 2005 Fifth International Conference on, 2005, IEEE, 73-76.

[22] P. Maragos, Pattern spectrum and multiscale shape representation. Pattern Analysis and Machine Intelligence, IEEE Transactions on, 1989, 11(7), 701-716.

[23] I. De, B. Chanda, and B. Chattopadhyay, Enhancing effective depth-of-field by image fusion using mathematical morphology. Image and Vision Computing, 2006, 24(12), 1278-1287.

[24] M. Oliveira and N. J. Leite, A multiscale directional operator and morphological tools for reconnecting broken ridges in fingerprint images. Pattern Recognition, 2008, 41(1), 367-377.

[25] H. R. Tizhoosh, Fuzzy image processing. Publisher: Springer-Verlag. Kartoniert (TB), Deutsch, 1997.