An improved mathematical model for assessing the performance of the SDHW systems

Imad A. Khatib imadk@ppu.edu
Renewable Energy and Environment Research Unit
Palestine Polytechnic University
Hebron, 19000, Palestinian Authority

Moh’d Awad mohdawad@ppu.edu
Renewable Energy and Environment Research Unit
Palestine Polytechnic University
Hebron, 19000, Palestinian Authority

Kathem Osaily kazema@ppu.edu
Renewable Energy and Environment Research Unit
Palestine Polytechnic University
Hebron, 19000, Palestinian Authority

Abstract

Short term testing of the SDHW system components is an essential prerequisite for assessing the system’s long-term thermal performance. Relevant performance parameters are therefore, crucial in recommending systems outputs and feasibility. Short term testing methods based on ANSI/ASHRAE and ISO standards testing methods and the mathematical system energy input/output models used showed sensitivity to variation of the day length used in testing and to different levels of irradiance. On the bases of the thermal performance testing carried out for a SDHW system and the outcomes of the day-by-day long-term performance prediction, a new mathematical system energy model is proposed which may reduce the number of tests required, minimizes the dependency on the day length and the sensitivity of the levels of irradiance.

Keywords: SDHW Systems, Performance Testing, Long-Term Performance Prediction, Energy Input/output System Equation model.

1. INTRODUCTION

The potential of utilizing solar energy in Palestinian areas is so promising. Since the 1970s people began to use solar energy for heating water for domestic uses and after 36 years; i.e. in 2006, survey studies done by the Palestinian Central Bureau of Statistics [1] indicated that more than 70% of Palestinian dwellings are equipped with solar domestic hot water (SDHW) systems. The SDHW systems are manufactured locally using traditional manufacturing processes that make these products very competitive and feasible when compared to other similar imported products. Until very recently, evaluating the performance of the locally manufactured SDHW systems could not be realized as there exists no official standards institute to regulate the process, and even after the establishment of the Palestinian Standards Institution (PSI) until recently the relevant standards for testing the performance of the SDHW systems are inactive.

The establishment of the Renewable Energy and Environment Research Unit (REERU) at the Palestine Polytechnic University and its involvement in a multilateral project funded by the US-Citizen Exchange Program of the Department of State entails its cooperation with the Florida Solar Energy Center (FSEC) of the University of Central Florida and a consortium of universities in the Middle East. The outcome of the cooperation is the establishment of the solar energy testing facility at REERU and strengthening the required human capacity. Through cooperation among REERU, the Palestinian Standards Institute and the Ministry of Higher Education, REERU was assigned to evaluate the performance of the locally produced SDHW systems and to validate a standard performance testing procedure that enables understanding, predicting and comparing the performance of the different SDHW systems.

2. TESTING SDHW SYSTEM'S COMPONENTS

The SDHW system used is a thermo-siphon integrated two flat-plate collectors tested with a thermal storage tank in an open cycle. The thermal storage tank capacity is 126-Liters. Each collector is having an aperture area of 1.42m^2. The collector housing is made of galvanized steel with 30mm polyurethane insulation separating the housing from the flow risers and headers tubes that are made of iron. The absorber plate is made of black steel and above it are transparent glass plates. This configuration is usually used in all locally manufactured flat plate solar collectors. The storage tank core is non-enameled made of 4mm steel sheets insulated from the housing with 50mm polyurethane and it withstands a maximum attainable pressure of 12bar. The collector housing is made of a 0.4mm galvanized steel sheet.

Both the flat plate collectors and the storage tank were tested under transient conditions in accordance with ANSI/ASHRAE 93-2003 [2] and ISO 1995 [3] standards testing procedures and methods.

2.1. TESTING FLAT-PLATE SOLAR COLLECTORS

The method used to test the collectors operating with constant flow rate is the steady-state method. The method was first proposed by Hill and Kausuda [4], and later published in ASHRAE standard [5] and ISO 1995 [3]. The steady state testing method depends on measuring the instantaneous collector efficiency over a range of operating temperature. The data collected should correspond to stationary conditions over a period of 15 to 20 minutes. A straight line presentation is then plotted using the relationship:

$$\eta = \eta_o - UT^* \qquad (1)$$

Where the reduced temperature difference T^* is computed using Eq. 2:

$$T^* = (T_m - T_a) / G_T \qquad (2)$$

And T_m is the mean plate operating temperature, i.e.

$$T_m = (T_i + T_e) / 2 \qquad (3)$$

The slope in Eq. 1 represents the collector heat loss coefficient; U and the intercept with the Y-axis (η_o) is the collector zero loss efficiency or the optical efficiency. For each testing point, the energy output of the collectors is calculated as:

$$Q_{out} = m_c\, c_P\, (T_e - T_i) \qquad (4)$$

and the energy input as

$$Q_{in} = G_T\, A_C \qquad (5)$$

The resulting of instantaneous efficiency η represented by the ratio of the output to the input (Q_{out}/Q_{in}) is plotted versus the reduced temperature difference T^* in the straight line presentation of Fig. 1, which represents the collector performance.

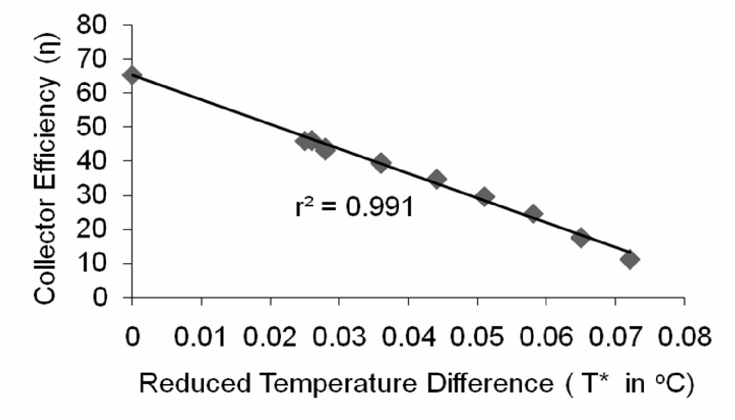

FIGURE 1: Solar collector testing results.

The results showed that the collectors have an optical efficiency (η_o) of 65%, which agrees well with optical efficiency values for similar collector types, and have a heat loss coefficient (U) of 7.2 $W/m^{2o}C$.

2.2. TESTING THERMAL STORAGE TANK

The thermal storage tank performance test is carried out to determine the storage heat loss coefficient UA_S. The test method is described in ASHRAE 93-2003. In this test, the storage tank is charged to a temperature 61.7 ^oC attained using the solar collectors and allowed to cool for 48 hours. During charging, no fluid is added to or extracted from the system. Hourly measurement is performed for the storage temperature and the ambient air temperature where the test is conducted.

By assuming a fully mixed storage losing heat to ambient temperature of T_a it is possible to integrate the storage tank energy balance equation (Eq. 6) over the test duration; i.e. *t* in seconds to compute the storage heat loss coefficient as in Eq. 7, i.e.

$$M_s\, c_p\, T_s = UA_s\,(T_a - T_s) \qquad (6)$$

$$UA_s = -1/t\; M_s\, c_p\, \ln\{1-(T_{sf} - T_{si}) / (T_a - T_{si})\} \qquad (7)$$

A plot (Fig. 2) showing the store temperature decrease over the testing time constitutes an indicator of the thermal storage performance.

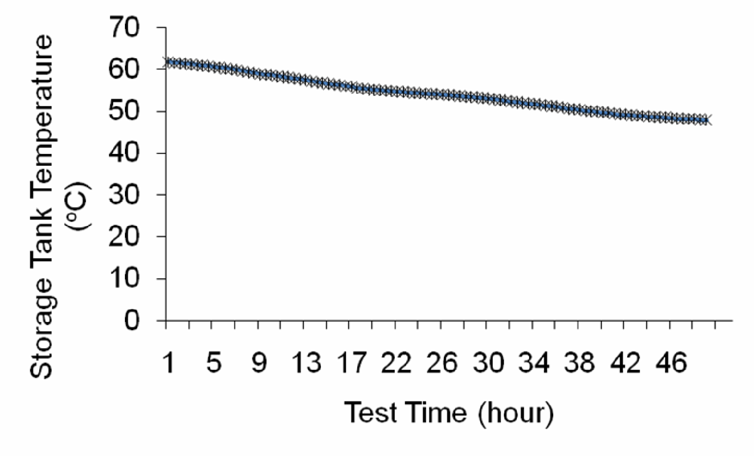

FIGURE 2: Storage tank temperature trend over the 48 hours.

The storage tank heat loss coefficient UA_S was computed at 1.42 W^oC^{-1}, a value considered relatively good for such thermal storage. However, in order to improve the thermal storage performance better insulation may be used including enameling the core of the storage tank.

2.3. TESTING IN SITU THE SDHW SYSTEM

It is clear that the ISO 9459-2 standard testing method is based on the European CSTG method [6]. The method assumes that for a fixed system and a fixed load, where there is little or no carry-over of energy from one day to the next, the climatic parameters which significantly affect daily system performance are:

- the daily solar irradiance on the plane of the collector H,
- the average ambient air temperature T_a during day time, and
- main cold water temperature supplied to the system T_C.

A correlation between these parameter is assumed which represents an approximate linear relationship between the solar irradiance and the system output which takes the form in Eq. 8:

$$Q_{out} = a_1H + a_2(T_a - T_c) + a_3 \qquad (8)$$

The correlation expressed in Eq. (8) forms the basis of the test method where, from results of several tests employing different values of H, T_a and T_c, the values a_1, a_2 and a_3 can be determined using a multi-regression fit method. It is worth mentioning that a_1 represent the irradiance part of the model equation, a_2 is the part of the energy lost or gained from the temperature difference and a_3 is the balance coefficient.

The test procedure carried out consists of one day tests which are independent of each other. For each test the system is first preconditioned bringing the store temperature to the cold water temperature. During that time collectors are covered to ensure no heat gained by solar radiation. The circulation of water through the store is stopped after ensuring that the temperature in the store is uniformly distributed. The system then charges by taking off the collectors' cover and exposing them to sun rays. Immediately after the charge phase a single draw-off of three times the store volume is performed in order to ensure that there was no carryover of energy to the next day.

The test is repeated over several days constituting different values of irradiance levels, ambient air temperatures, and mains water temperatures. During the course of each test the irradiance is measured over the plans of the collectors. And during the charging phase, hourly measurements are performed for the ambient air temperature, and the store temperature. The temperature difference between the outlet and the inlet to the store is also recorded over a time increment of 60 seconds.

The test is performed using an average flow rate of draw-off that is recommended by the testing method, which is 600 Liter/hour. The data obtained from the different tests are used in the mathematical thermal model equation (Eq. 8) and by using a multi-regression method the mathematical model coefficients a_1, a_2 and a_3 are identified for the different levels of irradiances (Table 1).

Irradiance Level	a1 (m^2)	a2 (MJ/K.day)	a3 (MJ/day)
Low	0.56	0.26	0.60
Medium	0.53	2.11	4.02
High	1.01	1.19	3.72

TABLE 1: Mathematical model constants (a-parameters).

The values of the model constants a_1 – a_3 are used then in predicting the long-term performance of the SDHW system.

2.4. SYSTEM DRAW-OFF TEST

The performance of the SDHW system depends on the internal operation of its component. One of the most influential parts of the SDHW system is the thermal storage tank. During extraction of energy the type of the store is dependent on the flow rate. The stratification will decrease with increase in the flow rate through the store due to increase in mixing between mains cold water entering the store and the hot water inside the store.

The system under test showed that its vertical storage tank type tends to be fully mixed during the energy charging phase. During extraction of energy the store appears to be stratified. The stratification increase with the increase in the energy input. In Fig. 3 below are the three curves showing different draw-offs under different irradiance levels. It could be seen that the discharged heat (Q_{out}) is proportional to the area under the curve.

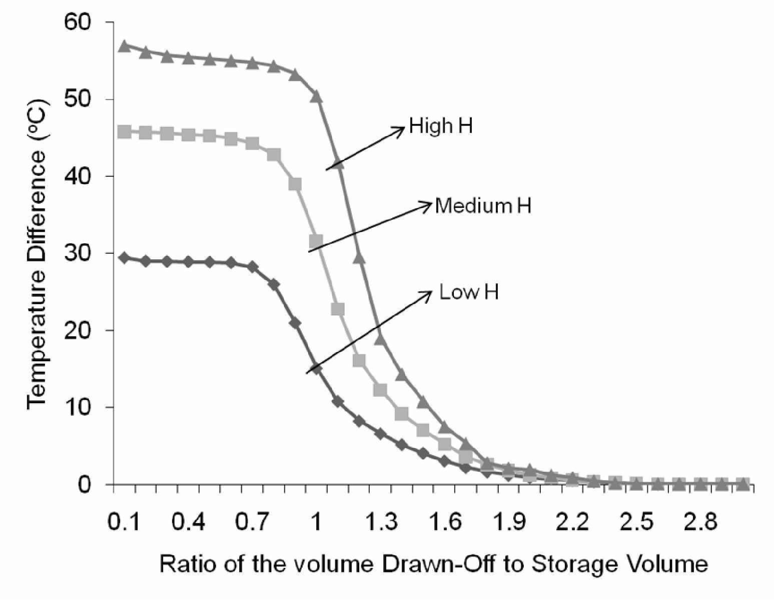

FIGURE 3: Draw-off curves for the different irradiance levels.

The draw-off curves that represent the energy output of the system when drawing 3 times its volume, could be used to measure the energy output from the system and the energy left over in the system by assigning the draw-off energy fraction f(v). By knowing the demand water volume that would be drawn-off (extracted) from the system, the fraction f(v) could be calculated. In this test a demand water volume of 150 Liter/day is considered, which constitute 1.19 times the tank volume of 126 Liter. Using the draw-off curves for the different levels of irradiance, the different values of f(v) could be calculated as shown in Table 2. The same table shows the fraction of energy when extracting the same tank volume of 126 Liter.

Irradiance Level	f(v) for 150 Liter	f(v) for 126 Liter
Low	0.78	0.66
Medium	0.79	0.68
High	0.84	0.72

TABLE 1: Values of the fraction f(v) calculated for different levels of Irradiance.

2.4. SYSTEM MIXING TEST

While the draw-off profile represents the effect of temperature stratification in the storage tank, and mixing during discharging the system, there is a need to determine the effect of mixing during draw-off independent of the temperature stratification inside the store. This is achieved by discharging the system after establishing a uniform temperature distribution inside the store, and by using the same flow rate utilized in draw-off profiles.

The resulting mixing profile can be seen in Fig. 4. It is seen that after consuming a volume equals to store volume (i.e. 126 Liter), the fraction of energy extracted g(v) is approximately 0.86 of the total energy withdrawn, and 0.90 of the total energy is for a demand load of 150 Liter.

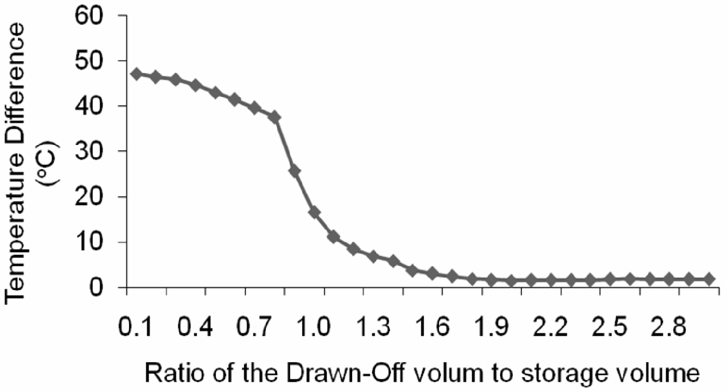

FIGURE 4: Mixing profile.

3. LONG-TERM SYSTEM PERFORMANCE PREDICTION

Long term performance prediction of the SDHW system is computed using a Day-by-Day method described in the European CSTG documents and later adopted by the ASHRAE 93-2003. The method requires several parameters to be pre-identified, i.e. those identified in the short-tem tests, as well as the dominant meteorological settings (or any other setting) for the location where the system is going to be installed. In addition daily mains cold water temperature recorded or calculated is considered essential for consistent calculations. In this respect, the British Standards (BS 5918) of 1982 [7] referenced a correlation for computing mains cold water temperature. This correlation was later modified to suit the Palestine climate and is written in the form:

$$T_c = 18 - 5.5 \times \cos((2 \times pi/365.5) \times (D + 11.25)) \quad (9)$$

For the long-term calculation, the annual climate data used is taken from the Meteorological Station of the Renewable Energy and Environment Research Unit (REERU).The weather station records values on time increment of 15 minutes and store them for each year.

The Day-by-Day method calculate the performance of the SDHW system for each day using the input meteorological values of that day, and taking into account any energy in the system that is carried over from the previous day. The method assumes that if the water in the storage tank is at a temperature higher than the cold water temperature at the beginning of the day, due to carry-over of energy from the previous day, this energy is equally spread over the tank volume at start of the next day (i.e. the storage tank is always in a fully mixed state with a uniform temperature at the start of each day).

The total energy captured by the first operating day is given by the Input\Output diagram. The draw-off temperature profile enables the division of this amount of energy into useful energy extracted during draw-off and energy carried over through using the fraction f(v).

The energy remaining in the storage tank will partly be lost overnight due to heat losses , and will force the system to start the next morning at a temperature higher than the cold supply temperature. At this stage, and for each of following days, the assumption mentioned before is made so that the energy is uniformly distributed over the store, leading to a uniform temperature at the beginning of the next day.

At the end of the second day and all the following days the system output can be calculated using the morning store temperature. A division is made in used and remaining energy using the draw-off profile f(v). This energy output represent only a part of the total energy output, as the system refilled with water at temperature T_c and not the morning store temperature. The division of the second part of the extracted and remaining energies is made based on the mixing-profile g(v). And so forth, the calculation is performed day-by-day, for each month and for the whole year. The output are the solar fraction for each month of the year (f_m), the yearly solar fraction (f), the energy output from the system and the energy delivered to the system.

3.1. LONG-TERM PREDICTION RESULTS

The system coefficients a_1, a_2 and a_3 computed are used in three consecutive model runs to predict the annual system performance. The annual delivered energy Q_{out}, which is the sum of the daily delivered energy, was calculated using a fixed demand volume of 150 Liters, and a demand temperature of 60 ^{o}C was chosen for the calculation.

The results of the annual performance prediction using the day-by-day performance prediction method is seen in Fig. 5 where both energy delivered to the SDHW system and energy extracted for each month of the year considered are presented in histograms.

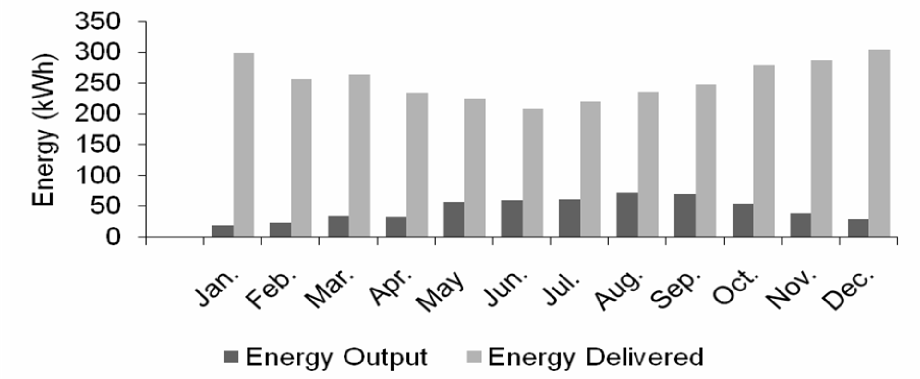

FIGURE 5: Histogram showing monthly variations between energy output and energy delivered.

The annaul solar fraction was computed at 0.18 which means in economic words an annual saving of more than US$120 and a payback period for the SDHW system of 6-7 years.

It should be mentioned here that the mathematical input/output model (Eq. 8) showed high sensitivity for the day length and the variation in the level of irradiance.

3.2. RECOMMEDNED MODIFIED MATHEMATICAL MODLE

The sub-task E of the International Energy Agency (IEA) has suggested a stationary model for the short-term testing of the SDHW systems [8]. The model was called the c-model that identifies 5 parameters, reduce the testing time and provide more variability of test data. The correlation model expressed in the form:

$$\begin{aligned} Q_{out}\,(1+c_3c_4\delta t / (M_L\, c_p)) &= c_1\, \Sigma\, (G_T\, K_{\alpha\tau} - c_2\, \{T_c - T_a + \\ &\quad c_4\, Q_{out} / (M_L\, c_p)\})^{+}\, \Delta t - c_3\, \delta t\, (T_c - T_a) - \\ &\quad c_5\, \delta t\, (T_D - T_a) \end{aligned} \qquad (10)$$

The coefficient $c_1 - c_5$ are system parameters that can be identified by a best fit to test data. It is worth mentioning that the last term of Eq. 10 accounts for thermal auxiliary source during testing and hence for real testing the term could be excluded; i.e. c_5 accounts to zero. Marshall et al. [9] have added a term to Eq. 10 that accounts for carryover of energy, changing the model from stationary to non-stationary model as in Eq. 11:

$$\begin{aligned} Q_{out}\,(1+c_3c_4\delta t / (M_L\, c_p)) &= c_1\, \Sigma\, (G_T\, K_{\alpha\tau} - c_2\, \{T_c - T_a + \\ &\quad c_4\, Q_{out} / (M_L\, c_p)\})^{+}\, \Delta t - c_3\, \delta t\, (T_c - T_a) - \\ &\quad M_s\, c_p\, \delta t\, (T_s^{n+1} - T_s^{n}) \end{aligned} \qquad (11)$$

Where, T_s^{n+1} and T_s^{n} are the store temperature at end of the test day and at the beginning of the test day respectively.

Eq. 11 could be written in terms of energy output (Q_{out}) as:

$$\begin{aligned} Q_{out} &= c_1\Sigma(G_T - c_2\, \{T_c - T_a + c_4\, Q_{out} / (M_L\, c_p)\})^{+}\, \Delta t - \\ &\quad c_3\, \delta t\, (T_c - T_a) - c_3\, c_4\, \delta t\, Q_{out} / M_L\, c_p) - \\ &\quad M_s\, c_p\, (T_s^{n+1} - T_s^{n}) \end{aligned} \qquad (12)$$

By assuming that all energy is utilizable (the + sign) then the equation should be reduced to:

$$\begin{aligned} Q_{out}\,(1+(c_1c_2c_4 / M_L\, c_p)\Sigma\Delta t + c_3c_4\, \delta t / M_L\, c_p) &= \\ &\quad c_1\, \Sigma G_T\, \Delta t - c_1c_2\, (T_c - T_a)\Sigma\Delta t - c_3\, \delta t\, (T_c - T_a) - \\ &\quad M_s\, c_p\, (T_s^{n+1} - T_n^{n}) \end{aligned} \qquad (13)$$

Or

$$\begin{aligned} Q_{out} &= [c_1 / (1+d+e)]H + \\ &\quad [c_1c_2\, \Sigma\Delta t + c_3\delta t\; / (1+d+e)]\, (T_a - T_c) + \\ &\quad [-1 / (1+d+e)]\, \Delta Q_{store} \end{aligned} \qquad (14)$$

Assuming that the values:
$d = (c_1c_2c_4 / M_L\, c_p)\, \Sigma\, \Delta t$
$e = c_3c_4\, \delta t / M_L\, c_p$
$\Delta Q_{store} = M_s\, c_p\, (T_s^{n+1} - T_s^{n})$

Comparing the model Eq. 14 with the model Eq. 8 showed that the coefficients $a_1 - a_3$ are given by:

$a_1 = c_1 / (1+d+e)$
$a_2 = c_1c_2\, \Sigma\, \delta t + c_3\, t / 1+d+e$
$a_3 = -1 / 1+d+e$

By knowing either $a_1 – a_3$ or $c_1 – c_4$ the other parameters could be identified. This mathematical model assumes one day length and the four c - parameters depend primarily on the SDHW system operation conditions and its performance parameters. Hence, to reduce the sensitivity of the input/output model (Eq. 8) on the day length and the irradiance levels it is possible to perform few short-term tests using constant day length and solve the equation to find c-parameters and the corresponding a-parameters for the long-term calculations.

The mathematical model needs to be verified and hence future work will concentrate on testing the model and comparing the results with the previous ones.

4. CONCLUSIONS

Testing the performance of the locally manufactured SDHW system and its components using the ASHRAE 93-2003 and ISO 1995 and combining the resulted performance parameters to the day-by-day long-term performance prediction procedure brought relatively good results. However, it is found that the results of the mathematical input/output model and the testing procedure are sensitive to day length and to irradiance levels. A new mathematical model that is based on the IEA utilizable model and the input/output model used is proposed. It is suggested that the new model may minimizes the dependency of the input/output model on the day length and the irradiance levels.

4. CONSLUSION & FUTURE WORK

5. REFERENCES

1. PCBS “Households energy survey results - April 2008”. Palestinian Central Bureau of Statistics, Palestinian Authority, 2008
2. ANSI/ASHRAE Standards 93-2003 “Methods of testing to determine the thermal performance of solar collectors”. ASHRAE publication, Atlanta, GA30329, 2003
3. ISO 1995 “Solar heating-domestic water heating systems – part 2: outdoor test method for system performance characterization and yearly performance prediction of solar only systems”. Ref. no. ISO 9459-2:1995(E), International Standards Organization, 2995
4. Hill, J.E. and Kausuda, T. “Methods of testing for rating solar collectors based on thermal performance”. NBS Report, NBSIR, 74-635, 1974
5. ASHRAE Standards 95-1981 “Methods of testing to determine the thermal performance of solar domestic water heating systems”. AHRAE publication, Atlanta, GA30329, 1981
6. European Solar and System Testing Group “Test method for solar water heating systems, Second international draft, Proceedings of the 6th meeting of the European Solar Collector and System Testing Group (CSTG), Joint Research Center, Ispra, Italy, 1987
7. BS 5918 “Code of practice for solar heating systems for domestic hot water”. British Standards Institution, London, UK, 1982
8. Spirikl, W. “SDHW short-term test simulation using a utilizability based correlation model”. Internal Report to the International Energy Agency (IEA), III-E Document, IEA, 1987
9. Marshall, R. and Barragan, F. A simulation study of the IEA system test procedure”. Dept. of Mechanical Engineering and Energy studies, report no. 1440, University of Wales, Cardiff, U, 1988