

# A Method of Constructing Balanced Repeated Measurement Designs for First Order Residual Effects in Information Security

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## Abstract

Very few papers in information security fields discuss repeated measurement designs and analysis. Balanced repeated measurement designs for first order residual effects are used to estimate both treatment and residual effects more precisely. The treatments for these effects can be types of security controls. In this paper we address the need of repeated measurement designs and propose a method of constructing them using both complete and incomplete block designs. This paper attempts to clear up the definition of Balanced repeated measurement designs for first order residual effects designs (called BRM1) first given by Williams and the definition (called BRMP) given by Patterson. Some properties are also discussed how to use them in practice. Further research will be conducted for minimal and optimal repeated measurement designs in the information security field.

**Keywords:** Balanced Repeated Measurement Design, Changeover Design, Carryover Design, Information Security, Balanced Incomplete Block Design.

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## 1. INTRODUCTION

A repeated measurement design, also referred to as crossover, changeover, carryover, switchover, with  $n$  subjects,  $t$  treatments and  $p$  periods, or RM ( $t, n, p$ ) design, is an experimental design in which  $n$  subjects (experimental units) can be used repeatedly by exposing them to a sequence of  $t$  different or identical treatments in  $p$  periods. The effect of the treatment being applied is called the direct effect, and the effect of the previous treatment is called the residual effect (or carryover effect). If a residual effect is incurred after  $i$  periods, then we call it the  $i^{\text{th}}$  order residual effect.

The simplest such design is that of two subjects and two treatments, A and B, in two periods. We can apply treatment A to subject 1 in period 1 and treatment B in period 2. For subject 2, we reverse the order of application, that is, we apply treatment B to subject 2 in period 1 and treatment A in period 2. In this case, we say that we apply sequence 1 (treatment A first, then treatment B) to subject 1 and apply sequence 2 (treatment B first, then treatment A) to subject 2. Since there are two treatments applied to each subject, this is a RM ( $t=2, n=2, p=2$ ) design.

Repeated measurement designs are often used in pharmacology (Manrai et al., 2020), animal science (Zhao et al., 2020), agriculture (Gaitán-Rossi et al., 2019) and technology (Fisher, 2019). The variation between the subjects can be assumed to be much larger than the variation within the individual subjects in the case where there are budget limitations such as subjects are expensive or limited that we need to apply more than one treatment to each subject, limited time to conduct the experiment or need to train subject over a long period of time and no serious

damaging effect on the subjects, finding out the effect of different treatment sequences, or residual effects. For example, repeated measurement designs were used in assessing COVID-19 vaccine efficacy (Follmann et al., 2020 and Ren et al., 2021).

However, repeated measurement designs are not used in technology correctly. An existing paper to assess different online games' user accessibility in data mining does not provide uniform network environments as periods to control the network environment variability (Godzinski et al., 2022).

Since each observation consists of cumulative effects, involving both direct and residual effects, the analysis of the repeated measurement design will be more complicated than that of the design without residual effects. There are three ways to make the analysis simple: (1) by inserting a washout period (called a rest period) between the treatment periods to obtain the estimates of the direct and/or the residual effects more precisely than those of any other repeated measurement designs (Tippey et al., 2015); (2) by adding a pre-period with  $t=p$  (Lucas, 1957) and with  $t>p$  (Patterson, 1950) to remove the non-orthogonality between direct and residual effects; (3) by using known balanced repeated measurement designs which will be discussed in the Section 2. In Section 3 we propose a new inductive method and construct balanced repeated measurement designs using all treatments in each subject and then in Section 4 we will construct balanced repeated measurement designs using the method proposed in Section 3. In Section 5 we will examine some properties of our designs proposed in Section 3. In Section 6 and 7 we will discuss how to use our designs in practice. And an algorithm of generating balanced repeated measurement designs is given in the Appendix.

## 2. LITERATURE REVIEW

In this section we will discuss the development of the balanced repeated measurement designs under the existence of the first order residual effects. We will also review the definitions of the balanced repeated measurement designs as used by different authors.

### 2.1 BRM1 Designs for First Order Residual Effects

The most commonly used RM ( $t, n, p$ ) designs are balanced designs for first order residual effects given by Williams (1949, 1950) and Heday et al. (1975). The definition is given in the Definition 2.1.1.

Definition 2.1.1. A RM ( $t, n, p$ ) design is said to be balanced, or a BRM1( $t, n, p$ ) design, with respect to sets of direct and first order residual effects, if (1) each treatment occurs  $m_1$  times in each period each treatment is preceded by every other treatment  $m_2$  times.

Williams called these designs balanced residual effects designs. BRM1( $t, n, p$ ) designs are constructed using Latin Squares which are defined in the Definition 2.1.2 (Kempthorne, 1952).

Definition 2.1.2. A Latin Square is an  $n \times n$  array filled with  $n$  different symbols, each occurring exactly once in each row and exactly once in each column (Hinkelmann et al., 2014).

Special Latin Squares called cyclic Latin Squares and reduced Latin Squares are given in Definition 2.1.3 and Definition 2.1.4 respectively by McKay et al. (2005, 2006).

Definition 2.1.3. A cyclic Latin Square is a Latin Square where all entries in each row are generated cyclically.

Definition 2.1.4. A reduced Latin Square (normalized or in standard form) is a Latin Square where both its first row and its first column are in their natural order.

We will define a semi-reduced Latin Square in Definition 2.1.5.

Definition 2.1.5. A semi-reduced Latin Square is a Latin Square where its first row is in its natural order.

For example, the design in Table 1 is a cyclic, semi-reduced Latin Square.

There are two types of balanced repeated measurement designs. The first type is using complete block designs proposed by Williams, where all treatments are applied on each subject (or blocks). The others are using incomplete block designs proposed by Peterson, where not all treatments are applied on each subject. Williams constructed a BRM1( $n, n, n$ ) design using one Latin Square when the number of treatments  $t$  is even as follows:

For  $t$  treatments we construct the first subject with  $t$  periods by entering successive number from integer 1 in every other period from period 1 to period  $t$  and reversing the direction once period  $t$  has been reached. The remaining  $p-1$  subjects can be constructed cyclically from the first subject.

For example, when  $n=t=p=6$ , the first subject can be applied to treatments (1,6,2,5,3,4), the second subject has treatments (2,1,3,6,4,5), ..., etc. The BRM1(6,6,6) design with  $m_1=1$ , and  $m_2=1$  can be shown in Table 1.

Period	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5	Subject 6
1	1	2	3	4	5	6
2	6	1	2	3	4	5
3	2	3	4	5	6	1
4	5	6	1	2	3	4
5	3	4	5	6	1	2
6	4	5	6	1	2	3

TABLE 1: BRM1(6, 6, 6) design.

When  $t$  is odd, he constructed a BRM1( $r, n=2t, p=t$ ) design with  $m_1=2$ , and  $m_2=2$  using two Latin Squares. For example, when  $n=3$ , a BRM1(3,6,3) can be shown in Table 2.

Period	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5	Subject 6
1	1	2	3	3	1	2
2	3	1	2	1	2	3
3	2	3	1	2	3	1

TABLE 2: BRM1(3, 6, 3) design.

The properties of Williams' BRM1 designs are: (1) all direct effects are estimated with equal precision; (2) all residual effects are estimated with equal precision. The advantages of Williams' BRM1 designs over unbalanced RM designs are: (1) the efficiency is increased (i.e., we can get more precise estimates of direct and residual effects than any unbalanced RM designs); (2) the design and analysis are simpler than those for other designs.

However, when  $p < t$ , we cannot use Williams' designs. Patterson used incomplete Latin Squares (called Generalized Youden Squares) to construct BRM1 designs, which are defined here as BRMP designs for such situations by Patterson (1950, 1951, 1952) as described in Section 2.2.

## 2.2 BRMP( $t, n, p$ ) Designs

Before defining BRMP designs, we need to define BIB (Balanced Incomplete Block) designs first. An incomplete block means not all treatments appear in any block.

Definition 2.2.1. A BIB design is an incomplete block design where all pairs of treatments appear in the same block the same number of times.

For example, the design in Table 3 using subjects S1-S12 as blocks is a BIB design where every pair of treatments appears together in 4 blocks. After deleting the last period, the resulting design is still a BIB design where every pair of treatments appear together in 2 blocks.

Period	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
1	1	2	3	4	1	2	3	4	1	2	3	4
2	2	1	4	3	3	4	1	2	4	3	2	1
3	3	4	1	2	4	3	2	1	2	1	4	3

TABLE 3: BIB design.

Definition 2.2.2. A BRMP( $t, n, p$ ) design is a BRM1( $t, n, p$ ) design which satisfies the following conditions: (1) the design is a BIB with subjects used as blocks; (2) deleting the last period, the design is still a BIB; (3) for every pair of treatments (A, B), the number of subjects in which B occurs when A is in the last period is the same as the number of subjects in which A occurs when B is in the last period.

For example, the design in Table 3 is a BRMP(4, 12, 3) design. It is also a BRM1(4, 12, 3) design with  $m_1=3$ , and  $m_2=2$ . However, the designs in Table 1 and 2 are not BIB designs, so they are not BRMP designs. On the other hand, a BRM1 design may not be a BRMP design. For example, the design in Table 4 is a BRM1 (5, 10, 3) design with  $m_1=2$  and  $m_2=1$  using subjects S1-S10 as blocks. However, it is not a BIB design where treatments 1 and 2 appear twice in the same block, whereas treatments 1 and 3 appear four times in the same block. Therefore, it is not a BRMP design.

Period	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
1	1	3	5	4	2	4	3	2	5	1
2	2	4	1	5	3	3	2	1	4	5
3	4	1	4	2	5	1	5	3	2	3

TABLE 4: BRM1(5, 10, 3) design.

From the examples above, the set of BRM1 designs and the set of BRMP designs are not the same. The set of BRMP designs is a proper subset of the set of BRM1 designs (Rosen, 2019). Both sets are not equal, in which the design in Table 4 is an example.

In Section 3 a new way of constructing BRM1( $t, n, p$ ) designs is proposed.

### 3. CONSTRUCTING BRM1 DESIGNS

In this section we will generate BRM1( $t, t, t$ ) designs when  $t$  is even. If  $t$  is odd, we can generate BRM1( $t, 2t, t$ ) designs. In all designs, rows represent periods, and all columns represent subjects.

Case 1. When  $t$  is even, generate BRM1( $t, t, t$ ) designs:

Firstly, we generate the first column using the following algorithm: Starting with 1 in the first row, we generate the first element in each even row by adding the number 1 to another integer (called difference) from 1 to  $t/2$ , respectively. And the sum is the number in each row. Secondly, if there are empty rows, we can generate the skipped rows from the bottom to the top by adding 1 to the difference from  $t/2+1$  to  $t-1$ . And the sum is the element in each row. After creating the first column, we can generate the rest of columns cyclically.

For example, when  $t=2$ ,  $t/2=1$ . So we have 1 in row 1 and 2 in row 2. The number of rows= $t=2$ . Since we have no skipped row, the first column is 1 2. Then generate the rest of column cyclically. The design can be shown below:

For  $t=2$ , a cyclic Latin Square, BRM1 (2, 2, 2) design:

1 2  
2 1

For  $t=4$ , a cyclic Latin Square, BRM1 (4, 4, 4) design:  
Starting with 1 in row 1,  $t/2=2$ . The elements in the first column is  
Row 2:  $1+1=2$   
Row 4:  $1+2=3$   
Row 3:  $1+3=4$

The resulting cyclic RM (4, 4, 4) design is  
1 2 3 4  
2 3 4 1  
4 1 2 3  
3 4 1 2

For  $t=6$ :  
Starting with 1 in row 1, the first elements in the first column are  
Row 2:  $1+1=2$   
Row 4:  $1+2=3$   
Row 6:  $1+3=4$   
Row 5:  $1+4=5$   
Row 3:  $1+5=6$

The resulting cyclic RM (4, 4, 4) design is  
1 2 3 4 5 6  
2 3 4 5 6 1  
6 1 2 3 4 5  
3 4 5 6 1 2  
5 6 1 2 3 4  
4 5 6 1 2 3

For  $t=8$ :  
Starting with 1 in row 1, we generate the first column:  
Row 2:  $1+1=2$   
Row 4:  $1+2=3$   
Row 6:  $1+3=4$   
Row 8:  $1+4=5$   
Row 7:  $1+5=6$   
Row 5:  $1+6=7$   
Row 3:  $1+7=8$

The resulting design is  
1 2 3 4 5 6 7 8  
2 3 4 5 6 7 8 1  
8 1 2 3 4 5 6 7  
3 4 5 6 7 8 1 2  
7 8 1 2 3 4 5 6  
4 5 6 7 8 1 2 3  
6 7 8 1 2 3 4 5  
5 6 7 8 1 2 3 4

$t=10$ , a BRM1(10, 10, 10) design is

1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	1
10	1	2	3	4	5	6	7	8	9
3	4	5	6	7	8	9	10	1	2

9	10	1	2	3	4	5	6	7	8
4	5	6	7	8	9	10	1	2	3
8	9	10	1	2	3	4	5	6	7
5	6	7	8	9	10	1	2	3	4
7	8	9	10	1	2	3	4	5	6
6	7	8	9	10	1	2	3	4	5

Case 2. When  $t$  is odd, generate  $BRM1(t, 2t, t)$  designs:

Firstly, we generate the first column using the following algorithm: Starting with 1 in the first row, we generate the first element in each even row by adding the number 1 to another integer (called difference) from 1 to  $\text{floor}(t/2)$ , respectively. And the sum is the number in each row. If difference reach to  $\text{floor}(t/2)$ , then use difference  $\text{floor}(t/2) + 1$  for the  $t$ th row. If there are missing rows, then reverse the direction, use difference  $\text{floor}(t/2)+2$  to  $t-1$  row from bottom up to fill in the rest of rows. After the first column has been generated, then we generate the rest of the Latin Square cyclically. Finally, we generate the first row of another Latin Square by using last row first and reverse the previous Latin Square row-wise. The resulting design is by combining those two Latin Square side by side.

For example, if  $t=3$ :  $\text{floor}(t/2)=1$

Firstly, we generate the first column in the following:

Starting with 1 in row 1,

Row 2:  $1+1=2$

Row 3:  $1+2=3$

The first cyclic Latin Square is

```
1 2 3
2 3 1
3 1 2
```

Secondly, we create the first row of another Latin Square starting from the bottom row of the first Latin Square resulting 3 1 2. Then the rest of rows coming from reversing the rest rows of the first Latin Square, the resulting Latin Square is

```
3 1 2
2 3 1
1 2 3
```

Put the second Latin Square along with the second one side by side resulting:

```
1 2 3 3 1 2
2 3 1 2 3 1
3 1 2 1 2 3
```

For  $t=5$ :  $\text{floor}(t/2)=2$

First, create a Latin Square using the method as described above:

Starting with 1 in row 1

The first column is generated below:

Row 2:  $1+1=2$

Row 4:  $1+2=3$

Row 5:  $1+3=4$

Row 3:  $1+4=5$

The first cyclic Latin Square is

```
1 2 3 4 5
```

2 3 4 5 1  
 5 1 2 3 4  
 3 4 5 1 2  
 4 5 1 2 3

Secondly, create another Latin Square from starting from the bottom row of the first Latin Square resulting:

4 5 1 2 3  
 3 4 5 1 2  
 5 1 2 3 4  
 2 3 4 5 1  
 1 2 3 4 5

Put the second Latin Square along with the second one side by side resulting:

1 2 3 4 5 4 5 1 2 3  
 2 3 4 5 1 3 4 5 1 2  
 5 1 2 3 4 5 1 2 3 4  
 3 4 5 1 2 2 3 4 5 1  
 4 5 1 2 3 1 2 3 4 5

In Section 4 we will construct BRM1(t,n,p) designs when  $p < t$ . The resulting designs are also BRMP designs.

#### 4. CONSTRUCTING BRMP DESIGNS

In this section if  $p < t$ , using the techniques in Section 3 we will generate BRM1(t,n,p) designs when p is even, where  $n = p \cdot C(t, p)$ . Note that  $C(t, p) = \frac{t!}{p!(t-p)!}$  is the number of p-combinations (Rosen, 2019). If p is odd, we can generate BRM1(t,n,p) designs where  $n = 2 \cdot p \cdot C(t, p)$ . In all designs, rows represent periods, and all columns represent subjects.

Case 1. When p is even,

There are  $C(t, p)$  numbers of different subjects in which p treatments are applied. For each of these different subjects each can create a BRM1(p,p,p) design by relabeling each treatment in order and using the method in Section 3. Therefore, we have total  $n = p \cdot C(t, p)$  subjects.

Example:

t=5, p=4

There are  $C(5,4)=5$  initial subjects of possible different treatments shown below:

1 1 1 1 2  
 2 2 2 3 3  
 3 3 4 4 4  
 4 5 5 5 5

Label treatments in order in each subject as a, b, c, and d, generate initial BRM1(4,4,4) (Latin Square) for each initial subject, and create five BRM1(4,4,4) designs side by side using the method in Section 3. The result is shown below:

1 2 3 4 1 2 3 5 1 2 4 5 1 3 4 5 2 3 4 5  
 2 3 4 1 2 3 5 1 2 4 5 1 3 4 5 1 3 4 5 2  
 4 1 2 3 5 1 2 3 5 1 2 4 5 1 3 4 5 2 3 4  
 3 4 1 2 3 5 1 2 4 5 1 2 4 5 1 3 4 5 2 3

This is a BRM1(5,20,4) design with  $m_1=4$ ,  $m_2=3$ . It is also a BRMP design.

Case 2. When t is odd:

There are  $C(t, p)$  numbers of different subjects in which  $p$  treatments are applied. For each of these different subjects each can create a  $BRM1(p, 2p, p)$  design by relabeling each treatment in order and using method in Section 3. Therefore, we have total  $n=2^p \cdot C(t, p)$  subjects.

Example:

$t=5, p=3$

There are  $C(5, 3)=10$  initial subjects of possible different treatments shown below:

1 1 1 1 1 1 2 2 2 3  
 2 2 2 3 3 4 3 3 4 4  
 3 4 5 4 5 5 4 5 5 5

Label treatments in order in each subject as a, b, and c, generate initial  $BRM1(3, 3, 3)$  (Latin Square) for each initial subject, and create ten  $BRM1(3, 3, 3)$  designs side by side using the method in Section 3. The result is shown below:

1 2 3 2 3 1 1 2 4 2 4 1 1 2 5 2 5 1 1 3 4 3 4 1 1 3 5 3 5 1 1 4 5 4 5 1 2 3 4 3 4 2 2 3 5 3 5 2  
 2 3 1 1 2 3 2 4 1 1 2 4 2 5 1 1 2 5 3 4 1 1 3 4 3 5 1 1 3 5 4 5 1 1 4 5 3 4 2 2 3 4 3 5 2 2 3 5  
 3 1 2 3 1 2 4 1 2 4 1 2 5 1 2 5 1 2 4 1 3 4 1 3 5 1 3 5 1 3 5 1 4 5 1 4 4 2 3 4 2 3 5 2 3 5 2 3

2 4 5 4 5 2 3 4 5 4 5 3  
 4 5 2 2 4 5 4 5 3 3 4 5  
 5 2 4 5 2 4 5 3 4 5 3 4

This is a  $BRM1(5, 60, 3)$  design with  $m_1=12, m_2=6$ . It is also a BRMP design.

Note that any balanced RM  $(t, n, p)$  designs satisfies that (1)  $n=m_1 t$  (2)  $n(p-1)=m_2 t(t-1)$ . The designs are minimal RM designs since  $m_1=1$  is the smallest integer that satisfies  $m_1(p-1)$  is congruent to 0 (mod  $t-1$ ) (Hedayat et al., 1973).

In section 5 we will describe some of the properties of the designs generated in Section 3.

### 5. PROPERTIES OF CYCLIC $BRM1(T, T, T)$ DESIGNS WHEN T IS EVEN

The  $BRM1(t, t, t)$  designs generated in Section 3 when  $t$  is even have properties shown in Table 5:

t	Number of different Cyclic Latin Squares= t!	Number of $BRM1$ cyclic designs
2	2	2
4	24	8
6	720	24
8	40320	192
10	3628800	2880

TABLE 5: Number of  $BRM1(t, t, t)$  designs.

The  $BRM1(t, n, p)$  designs generated in Section 4 when  $t > p$  have properties stated in Theorem 5.1.1 and 5.1.2.

Theorem 5.1.1. If  $t > p$  and  $p > 1$ ,  $BRM1(t, n, p)$  designs generated in Section 4 when  $p$  is even have properties that  $n=p \cdot C(t, p)$  with  $m_1=C(t-1, p-1)$  and  $m_2=C(t-2, t-p)$ .

Proof: It is obvious that the number of times that every treatment appears in each period are the same. Therefore,  $m_1 =$  number of any treatment that appear in all initial subjects =  $C(t-1, p-1)$ .



In addition, since every pair of treatments in the Latin Square generated by each initial subject appears exactly once,  $m_2 = \text{number of any pair of treatments in the same initial subjects} = C(t-2, t-p)$ .

Theorem 5.1.2. If  $t > p$  and  $p > 1$ ,  $BRM1(t, n, p)$  designs generated in Section 4 when  $p$  is odd have properties that  $n = p * C(t, p)$  with  $m_1 = 2 * C(t-1, p-1)$  and  $m_2 = 2 * C(t-2, t-p)$ .

Proof: The proof is like that of Theorem 5.1.1. Since when  $p$  is odd, we generate two Latin Squares for each initial subject. Therefore, both  $m_1$  and  $m_2$  are double of the results in Theorem 5.1.1, respectively.

Example. For the designs in Section 4, using Theorem 5.1.1 when  $t=5$  and  $p=4$ ,  $m_1 = 4!/(3!1!) = 4$ ,  $m_2 = 3!/(1!2!) = 3$ . When  $t=5$  and  $p=3$ , using Theorem 5.1.2,  $m_1 = 2 * 4!/(2!2!) = 12$  and  $m_2 = 2 * 3!/(2!1!) = 6$ .

Theorem 5.1.3. The  $BRM1(t, n, p)$  designs generated in Section 4 are also  $BRMP(t, n, p)$  designs.

Proof: Since all pairs of treatments appear in the same subject the same number of times in each initial Latin Square(s) and each subject is an incomplete block, the design is BIB. After deleting the last period for each initial Latin Square, they are also BIBs. Therefore, after deleting the last period for the entire design, it is still BIB. Since each initial Latin Square has the property that for every pair of treatments (A, B), the number of subjects in which B occurs when A is in the last period is the same as the number of subjects in which A occurs when B is in the last period. Therefore, The  $BRM1(t, n, p)$  designs generated in Section 4 are also  $BRMP(t, n, p)$  designs.

In Section 6 we show how  $BRM1$  designs help managers to manage the limited situations in practice.

## 6. MANAGERIAL IMPLICATIONS

The proposed design requires some management planning in practice. Under the situation that we want to apply all treatments in each subject, and if  $t=p$ , then we can use  $BRM1(t, t, t)$  designs if  $t$  is even. And if  $t$  is odd, we can use  $BRM1(t, 2t, t)$  designs. If we have a limited number of subjects  $n$  that are even, then  $t$  can be set to  $n$  and we can use a  $BRM1(n, n, n)$  design. On the other hand, if  $n$  is even and  $n/2$  is odd, then  $t$  can be set to  $n/2$ , and we use a  $BRM1(n/2, n, n/2)$  design. Otherwise, we may not use all subjects in the experiment.

If we choose not to apply all treatments in each subject, i.e.  $t > p$ , and if  $p$  is even, we can use  $BRM1(t, p * C(t, p), p)$  designs. On the other hand, if  $p$  is odd, then we can use  $BRM1(t, 2p * C(t, p), p)$  designs. Since  $(t-p)! \geq (p-p)! = 1$ , if  $p$  is even,  $n = p * C(t, p) \leq p * t! / p! = t! / (p-1)!$ . In addition,  $n = p * C(t, p) \geq p * p! / \{p!(t-p)!\} = p / (t-p)!$ . In other words, if  $p$  is even,  $t! / (p-1)! \geq n \geq p / (t-p)!$ . And if  $p$  is odd,  $2 * t! / (p-1)! \geq n \geq 2 * p / (t-p)!$ . For example, if  $t=5$  and  $p=3$ , then  $120 \geq n \geq 3$ . If  $t=5$  and  $p=4$ , then  $20 \geq n \geq 4$ . These restrictions for  $n$  can give a rough estimate of how many subjects needed to be used in  $BRM1$  designs. We will summarize the results in Section 7.

## 7. RESEARCH RESULTS AND CONCLUSION

In practice, we can apply the RM designs to measure the direct and residual effects on different security defense measures. In an information field, when we want to collect data for analysis, RM designs can be used for many models such as in papers (Shing et al., 2012) (Ismail et al., 2013) (Yasin et al., 2016). For example, we can compare the effects of applying firewalls on a network during a fixed period, and then subnetting network in the other period within an organization. And then vice versa. The subjects of the  $BRM1$  design are the two networks in an organization. The treatments are firewalls against outsiders and subnetting. And the observations are the number of successful hackings into the mentioned network from outside. Not only can we find out what each

security defense's effect, but we can see whether we should apply firewalls first or subnetting first to a network.

In this paper we have clarified the definition BRM1 designs first given by Williams and the definition BRMP designs given by Patterson; they are deductive. We propose an inductive method of constructing RM designs. We can get more precise estimates for both treatments and first order residual effects. However, if the number of treatments is larger than the number of periods, we prefer to use BRMP designs. When we use BRMP designs that are not BRM1 designs, we cannot estimate first order residual effects as precisely as treatment effects, which may be our main concern in practice. This is the reason why we want to construct BRMP designs which are also BRM1 designs. Unfortunately, the number of subjects required for the construction proposed grows fast as the number of treatments grow. A minimal balanced RM (t, 2t, p) design with  $p < t$  exists whenever t is a prime power (Hedayat et al., 1973). The optimal RM designs were investigated by Hedayat et al (1978) and Stufken(1991).

## 8. APPENDIX

Algorithm of Generating BRM1(t,t,t) designs:

- Create original index string str[] to be permuted
- Find all permutations of str[] and store them as a string called resultStr
- Create one Latin Square
- Find the precedence relation of each column in all possible Latin Squares
- Create other Latin based on permutation of rows
- Count precedence of all columns in each Latin square
- Sort precedence
- Find unique precedence pair size and determine balanced Latin square

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## 10. REFERENCES

- Fisher, P. (2019). Does repeated measurement improve income data quality? *Oxford Bulletin of Economics and Statistics*, 81(5), 989-1011.
- Follmann, D. et al. (2020). Assessing durability of vaccine effect following blinded crossover in COVID-19 vaccine efficacy trials. *NIH*. <https://pubmed.ncbi.nlm.nih.gov/33336213>.
- Gaitán-Rossi, P. et al. (2020). Food insecurity measurement and prevalence estimates during the COVID-19 pandemic in a repeated cross-sectional survey in Mexico. *Cambridge University Press*, 1-10.
- Godzinski, A., Stroinski, A., Piatek, W. and Stroinski, A. (2022). Pattern recognition in games using process mining. *International Symposium on Electrical, Electronics and Information Engineering*, 42-45.
- Hedayat, A. and Afsarinejad, K. (1973). Repeated measurement designs, I. *International Symposium on Statistical Design and Linear Models*. <https://ani.stat.fsu.edu/techreports/M261.pdf>.
- Hedayat, A. and Afsarinejad, K. (1975). A survey of statistical design and linear models. *Repeated Measurement Designs*, 1, 229 – 242.
- Hedayat, A. and Afsarinejad, K. (1978). Repeated measurement designs, II". *The Annals of Statistics*, 6(3), 619-628.
- Hinkelmann, K. and Kempthorne, O. (2014). Design and analysis of experiments: Introduction to experimental design. *Wiley*.

- Kempthorne, O. (1952). Design and analysis of experiments. *Robert E. Krieger*, 1952.
- Lucas, H. (1957) "Extra-period Latin square change-over designs", *Journal of Dairy science*, 40, pp. 225-239.
- Manrai, A. and Mandl, K. (2020) Escaping the COVID-19 testing paradox. *SSRN*. [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=3612991](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3612991).
- McKay, H. and Wanless, I. (2005). On the number of Latin squares. *Annals of Combinatorics*, 9, 335-344.
- McKay, H., McLeod, J. and Wanless, I. (2006). The number of transversals in a Latin square. *Des Codes Crypt*.
- Patterson, H. (1950). The analysis of change-over trials. *Journal of Agricultural Science*, 40, 375-380.
- Patterson, H. (1951). Change-over trials. *Journal of the Statistical Society, B(13)*, 256-271.
- Patterson, H. (1952). The construction of balanced designs for experiments involving sequences of treatments. *Biometrika*, 39, 32-48.
- Patterson, H. and Lucas, H. (1950). Extra-period change-over designs. *Biometrics*, 15(1), 116-132.
- Ren, M. et al. (2021). Contribution of temperature increase to restrain the transmission of COVID-19. *Innovation*, 2(1), 1-8.
- Rosen, K. (2019). Discrete mathematics and its applications, *McGraw-Hill*.
- Shing, M. et al. (2012). Analysis of n category privacy models. *International Journal of Computer Science and Security*, 6(5), 342-358.
- Stufken, J. (1991). Some families of optimal and efficient repeated measurements designs. *Journal of Statistical Planning and Inference*, 27(1), 75-83.
- Tippey, K., Ritchey, P. and Ferris, T. (2015). Crossover-repeated measures designs: Clarifying common misconceptions for a valuable human factors statistical technique. *Proceedings of the Human Factors and ergonomics Society Annual Meeting*, 59(1), 342-346.
- Williams, E. (1949). Experimental designs balanced for the estimation of residual effects of treatments. *Australian Journal of Science Research*, A(2), 149-168.
- Williams, E. (1950). Experimental designs balanced for pairs of residual effects. *Australian Journal of Science Research*, A(3), 351-363.
- Yasin, A. and Nasra, I. (2016). Dynamic multi levels Java code obfuscation technique. *International Journal of Computer Science and Security*, 10(4), 140-160.
- Zhao, J., et al. (2019). Reporting and analysis of repeated measurements in preclinical animals experiments. *PLOS ONE*, 14(8). <https://doi.org/10.1371/journal.pone.0220879>.
- Ismail, M. et al. (2013). New framework to detect and prevent denial of service attack in cloud computing environment. *International Journal of Computer Science and Security*, 6(4), 226-237.