# Vehicle Routing Problem in real case application

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**Abstract**

The delivery of goods from a warehouse to local customers is important and practical problem of supply chain management. Current customer demand changes from day to day. If the total demand is greater than the whole capacity of the internal fleet, the use of the external carrier becomes necessary. Thus; it may be more economical to use an external carrier instead of an internal vehicle to serve one or very few customers. In this work we attempt to model the company problem. We first try to assign vehicles to warehouse and determine the private fleet size needed at each one by taking into account the interventions of preventive maintenance. Then we develop a second model that routes this limited truck from two central warehouses to customers' with the option of using an external carrier.

**Keywords:** Vehicle Routing Problem, preventive maintenance, 3PL, external carrier.

## 1. INTRODUCTION

In industrial companies the optimization of the transport operations are essential to forward the goods to the final customers under the best conditions of cost and time. Several companies prefer to deal with third party carriers (3PL) instead of having their own trucks. The decision of outsourcing or not the distribution activities is a major strategic decision in a competing environment where customer's satisfaction with the lower cost is a must. In this work we consider a real case in which vehicles have limited capacity and must satisfy the request for a certain number of customers with the possibility of using external carriers. We attempt to develop a mathematical model adapted to the company constraints

## 2. LITERATURE SURVEY

For a few years, a new type of the vehicle routing problem has showing a growing interest in the routing problem with limited fleet, called m-Vehicle Routing Problem (m-VRP); The fleet is thus fixed, i.e. we don't consider the possibility to the company of varying the composition of the vehicle fleet. This aspect of the problem has been recently treated by the following authors: Genderau et al (1999), Taillard (1999), Wassam et Osman (2002), Lau et al (2003), and Tarantilis et al (2004).
The literature dealing with vehicle routing problems when external carrier services are available is relatively limited, in the work of Ball et al (1983) which deals with the distribution problem of chemical company. The problem consists in delivering several chemical products to customers. The used approach allows the problem to be conceptualized as a standard vehicle routing problem where one of the vehicles, which represents the common carrier, has significantly different costs and operating characteristics. A single-vehicle version of this problem was mentioned by Volgenant and Jonker (1987) who showed that the problem implying an internal fleet with only one vehicle and the external carrier can be transformed into a Traveling Salesman Problem. This problem was later studied by Diaby and Rmesh (1995) whose objective was to decide which customers should be visited by external carrier and to optimize the route of the remaining customers. The authors optimality solved this problem by using a branch and bound algorithm for a problem with up to 200 customers; The results shows that the proposed algorithm is efficient and viable for solving problems of medium to large size. A variant of the problem was addressed by Klincewicz, Luss and Polcher (1990) in a context where customer location change from day to day, they divided the geographic area into sectors and decide how best to serve each sector. The model determines the private fleet size and the specific assignment of each sector to a private fleet or to an external carrier. Chu (2005) addressed the problem of routing a fixed number of vehicles with limited capacity with the option of using an external carrier if the total demand is greater than the whole capacity of internal fleet. The author developed the mathematical model for the problem and solved it heuristically with a saved based heuristic construction, followed by intra-route and inter-route customers' exchanges. Bolduc, Renaud and Boctor (2006) performed Chu's results by using two different initial solutions that are then improved with more sophisticated customer exchanges. Better results are reported by bolduc et al (2007) on a set of benchmark instances with vehicle fleet that are either homogeneous or heterogeneous. A perturbation metaheuristic combines a local descent based on different neighborhood structures with two diversification strategies, namely a randomized construction procedure and a perturbation mechanism where a number of pairs of customers are swapped. Finally Coté and Potvin (2009) performed the solutions obtained by bolduc et al (2008) for homogeneous instance by proposing a tabu search heuristic.

## 3. PROBLEM DEFINITION

To remain closer to the real applications and to be able to emphasize some of all their complexities, we choose an application of reference specialized in the conception of routes as well as the distribution of the products of its two production units. These two units are specialized in the manufacturing of two types of products (AC) and (MVC). The company should plan, manage and construct the routes for its various production units. Our interest in the first part would be to assign the vehicles to the production units and then we will try to model a VRPPC with two depots and two types of products. The studied network contains a warehouse which lays out all the vehicles of two production units. The internal fleet is composed of a limited number of vehicles of heterogeneous type but homogeneous in capacity. We also distinguish products of various types which must be transported by using the appropriate type of vehicles. Each customer must be served only once and if the vehicles capacities are not sufficient to serve all the customers, the company will use a 3PL carrier. Our contribution mainly consists in proposing an operational approach to help the persons in charge to make the decisions which refer to the vehicles assignment to the warehouses and the development of the routes (Fig 1).

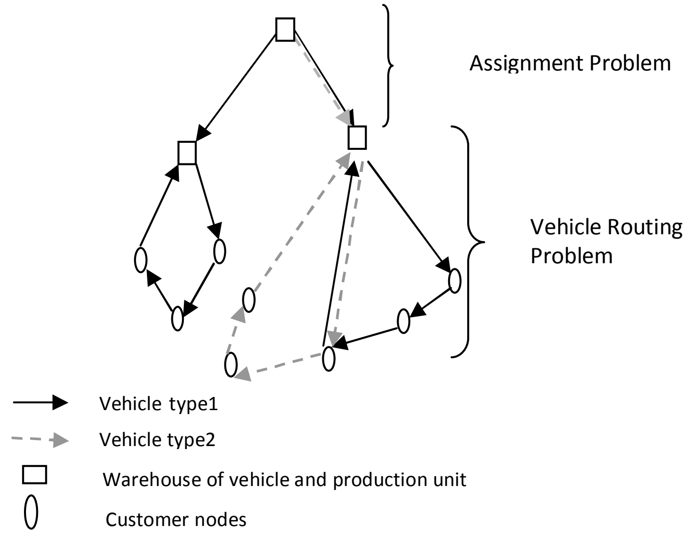


**FIGURE 1:** Studied network .

## 4. MODELING THE REAL CASE PROBLEM

Our contribution mainly consists in proposing an operational approach to help the persons in charge to make the decisions which refer to the vehicles assignment to the warehouses and the development of the routes.

### 4.1 Modeling the Assignment Problem

At the beginning, the number of trucks is unknown. In certain situations, it is possible that the number of vehicles affected by the distributer is not the optimal number of the trucks. Indeed, it is possible that the addition of a truck is advantageous in certain situations just like the withdrawal of another. Thus, in the worst cases, it will be necessary to use as many trucks as goods being served to satisfy customers demand. For example, if ten forwarding are to be planned, that will imply ten trucks to the maximum. Consequently, the number of the evaluated possibilities is important. This makes the problem very complex and implies a too many possibilities to be solved in a reasonable time with Ilog Cplex.

In a general way, the problem consists in assigning the vehicles to the warehouses in order to make it profitable for maximum use of the vehicles then to make sure that the products are delivered effectively.

**Assumptions**

- At the beginning of the day all the vehicles are available at the warehouse.
- The number of trucks for each available type is known.
- The number of customers and the each warehouse are known.

**Notations**

$K = 1 \ldots s$: Number of the vehicles of type 1.
$K = s+1 \ldots m$: Number of the vehicles of type 2.
$d_{i1t}$: A warehouse demdan of product type 1.
$d_{i2t}$: A warehouse demand of product type 2.
$Q_k$: Capacity of the vehicle k.
$C_{ik}$: Cost of assignment vehicle k to warehouse i.

$$v_{kt} = \begin{cases} 1 \text{ if the vehicle k} \in \text{ to the combinastion of the days in which k can be} \\ \text{in maintenance} \\ 0 \text{ otherwise} \end{cases}$$

*Index*

i: Warehouse index.
p: Product index.
k: Vehicle index.
t: Day index.

*Decision Variables*

Our first formulation uses the following variables:

$$a_{ikt} = \begin{cases} 1 \text{ if the vehicle k is affected to depot i at day t} \\ 0 \text{ otherwise} \end{cases}$$

**Mathematical Model**

$$Min \sum_{i=1}^{2} \sum_{k=1}^{m} \sum_{t=1}^{2} c_{ik} a_{ikt}$$

Subject to

$$\sum_{i=1}^{2} a_{ikt} \leq 1 \qquad (k = 1....m, t = 1, 2) \qquad (1)$$

$$\sum_{k=1}^{s} Q_k a_{ikt} \geq di1t \qquad (i = 1.2, t = 1, 2) \qquad (2)$$

$$\sum_{k=s+1}^{m} Q_k a_{ikt} \geq d_{i2t} \qquad (i = 1, 2, t = 1, 2) \qquad (3)$$

$$a_{ikt} \in \{0, 1\}$$

The objective function determines the minimal cost for each type of truck used for each warehouse. The constraints (1) impose that each vehicle must be affected only once to only one warehouse. The constraints (2) and (3) are associated with the conditions aiming so that the request for each type of product is satisfied.

**4.2 Integration of maintenance in the assignment model**

The purpose of the preventive maintenance is to anticipate the malfunction of the equipment and to prolong their lifespan. The companies are sensitive to these maintenance finalities; in order to ensure the best production performance level. Maintenance is essential for good performance of the vehicles. Consequently, in a transport system, we must maintain operational conditions of the vehicles fleets. Indeed, a late vehicle can involve expensive nuisances in time and loss of customers to the company. These vehicles will be subjected to maintenance actions, (preventive and corrective), in order to preserve them in operating condition.

In the literature the authors always assume that these vehicles are always available to satisfy customer's requests, but in the practical case the vehicles can be unreliable due to breakdowns or preventive maintenance actions. In this test, we will propose a new assignment model by taking into account these preventive maintenance actions, as maintenance is generally done at periods known in advance.

We kept all the assumptions and data of the above model and we will add the following assumptions and decision variables:

- A vehicle can be available or in maintenance.

$$m_{kt} = \begin{cases} 1 \text{ if the vehicle k is in maintenance the day t} \\ 0 \text{ otherwise} \end{cases}$$

We add the following constraints:

$$m_{kt} + \sum_{i=1}^{2} a_{ikt} \le 1 \qquad k = 1\ldots m,\ t = 1, 2) \qquad (4)$$

$$\sum_{t=1}^{2} m_{kt} \le 1 \qquad (k = 1\ldots m) \qquad (5)$$

$$m_{kt} \le v_{kt} \qquad (k = 1, \ldots, m,\ t = 1, 2) \qquad (6)$$

$$\sum_{t=1}^{2} m_{kt} \ge v_{kt} \qquad k = 1, \ldots, m,\ t = 1, 2) \qquad (7)$$

$$m_{kt} \in \{0, 1\}$$

$$a_{ikt} \in \{0, 1\}$$

The constraint (4) prohibits the use of the vehicle at the day t if it is in maintenance. The constraint (5) imposes that only one day of maintenance is selected for each vehicle. The constraints (6) and (7) force the vehicle K to be in maintenance only in the combinations of the corresponding days.

### 4.3 Modeling Vehicle Routing Problem

Let G=(N,A) be the graph where N={0…..n+1} is the vector set A={(i,j);i,j□N;i#j} is the arc set. The index 0 and n+1 are respectively the warehouses 1 and 2. A private fleet of m vehicles is available at warehouse 1: k={1…m} and of k vehicles are available at the warehouse 2: k={m+1……k}. These vehicles are of two types, the first type contains v vehicle: V= {1….v} and the second type contains s vehicles S = {1….s}. The fixed cost of vehicle k in the warehouse i is denoted by fik , its capacity by Qk and the customer demand j of product p is denoted by Qjp . A travel cost matrix (Cijk) is defined in A. Each customer can be served by a vehicle of the private fleet or by a 3PL carrier at a cost equal to zi. The VRPPC with two warehouses and two types of products consists in serving all customers in such a way that:

- The number of vehicles as their home bases of depart is fixed by the assignment model.
- Each customer can be served either by the private fleet vehicle or by a 3PL carrier.
- Some customers who can be served twice (by two types of distinct vehicles).
- Each vehicle leaves and returns to the deposit to which it was assigned.
- The total demand of any route does not exceed the capacity of the vehicle assigned to it.
- The objective is to minimize the sum of the 3PL carrier costs, the variable and the fixed costs of the internal fleet:

Our first formulation uses the following variables:

$$X_{ijk} = \begin{cases} 1 \text{ if vehicle k visits a vertex j immediately after vertex i} \\ 0 \text{ otherwise} \end{cases}$$

$$Y_{jkp} = \begin{cases} 1 \text{ if customer j receive product p by vehicle k} \\ 0 \text{ otherwise} \end{cases}$$

$$L_{jp} = \begin{cases} 1 \text{ if product p is transported to customer j by external carrier} \\ 0 \text{ otherwise} \end{cases}$$

The objective is to minimize the sum of external carrier cost, variable and fixed cost of the internal fleet:

$$Min \sum_{k=1}^{m} \sum_{p=1}^{2} f_{0k} Y_{0kp} + \sum_{k=m+1}^{K} \sum_{p=1}^{2} f_{n+1k} Y_{(n+1)kp}$$
$$+ \sum_{k=1}^{K} \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} C_{ij} X_{ijk} + \sum_{j=1}^{n} \sum_{p=1}^{2} C_{j} L_{jp}$$

The order of the customers visited by the vehicles must be specified. The problems approached imply the use of two warehouses. Consequently, each truck available must leave the warehouse to which it was affected. This can be written in the following way:

$$X_{(n+1)jk} = 0 \qquad (k = 1...m,\ j = 1...n) \qquad (1)$$
$$X_{0jk} = 0 \qquad (k = m+1...K,\ j = 1...n) \qquad (2)$$
$$X_{0jk} = X_{0jk} \qquad (k = 1...m,\ j = 1...n) \qquad (3)$$
$$X_{(n+1)jk} = X_{j(n+1)k} \qquad (k = m+1...K,\ j = 1...n) \qquad (4)$$

Once the vehicle k leaves the warehouse, it will have to deliver all containing goods. With this intention, it must obligatorily arrive at a destination (node h), discharge the goods and set out again to go to the following node. Each stage of arrival and starting to a node h is illustrated as follows:

$$\sum_{i=0}^{n+1} X_{ihk} = \sum_{i=0}^{n+1} X_{hjk} = Y_{hkp} \qquad (h = 0....n+1,\ k = 1....K \quad p = 1,2) \qquad (5)$$

The constraint (6) and (7) consists in sending at maximum two vehicles of various types: k ∊ V and k ∊ S to the same customer:

$$\sum_{k=1}^{v} \sum_{i=0}^{n+1} X_{ijk} \le 1 \qquad (\mathrm{j} = 1...n) \qquad (6)$$

The constraint (6) specifies that a customer who demands a product of type 1 can appear only on routes which use vehicles of the type 1.

$$\sum_{k=1}^{s} \sum_{i=0}^{n+1} X_{ijk} \le 1 \qquad (\mathrm{j} = 1...n) \qquad (7)$$

The constraint (7) specifies that a customer who demands a product of type2 can appear only on routes which use vehicles of the type 2.

$$Y_{jkp} - \sum_{i=0}^{n+1} X_{ijk} \le 0 \qquad \forall j = 1...n,\ k = 1....v,\ p = 1 \qquad (8)$$
$$Y_{jkp} - \sum_{i=0}^{n+1} X_{ijk} \le 0 \qquad \forall j = 1...n,\ k = 1....s,\ p = 2 \qquad (9)$$

The constraints (8) and (9) are used to force $Y_{jkp}$ to zero if the vehicle k does not visit the customer j.

$$\sum_{i=0}^{n+1} X_{ijk} + Y_{jkp} + L_{jp} = 0 \qquad (q_{j1} = 0,\ j = 1...n,\ k = 1....v,\ p = 1) \qquad (10)$$
$$\sum_{i=0}^{n+1} X_{ijk} + Y_{jkp} + L_{jp} = 0 \qquad (q_{j2} = 0,\ j = 1...n,\ k = 1....s,\ p = 2) \qquad (11)$$

The constraints (10) and (11) are the constraints which prohibit the visit to the customer j if he does not demand a product p.

$$\sum_{k=1}^{v} Y_{jkp} + L_{jp} = 1 \qquad (j = 1 \ldots n,\ q_{j1} \neq 0,\ p = 1) \qquad (12)$$

$$\sum_{k=1}^{s} Y_{jkp} + L_{jp} = 1 \qquad (j = 1 \ldots n,\ q_{j2} \neq 0,\ p = 2) \qquad (13)$$

The constraints (12) and (13) specify that each customer J is visited only once either by the 3PLcarrier or by their own fleet.

$$\sum_{j=0}^{n} q_{jp} Y_{jkp} \leq Q_k \qquad (k = 1 \ldots.. v,\ p = 1) \qquad (14)$$

$$\sum_{j=1}^{n} q_{jp} Y_{jkp} \leq Q_k \qquad (k = 1 \ldots.. s,\ p = 2) \qquad (15)$$

Constraints (14) and (15) ensure that the vehicle capacity is never exceeded.

$$\mu_i - \mu_j + (n+1) X_{ijk} \leq n \qquad (i = 1 \ldots. n,\ j = 1 \ldots n,\ k = 1 \ldots. K) \qquad (16)$$

Constraints (16) eliminate sub tours. This formulation is drawn from the article of Miller, Tucker and Zemlin (1960).

## 5. COMPUTATIONAL RESULTS

This section aims at having the results of the tests in order to validate if our formulation generates an acceptable solution in short time while using Cplex software. The validation is however constraint by the complexity of the problem. Consequently, the tests were made on small size problems, i.e. problems being able to be solved in an exact way in a reasonable time.
The data used to carry out these tests were provided by the company. We have all useful information for one working day: the number of vehicles, their capacities, the fixed costs, the delivery address and their requests.

| | | | t1 | t2 |
|---|---|---|---|---|
| Problem 1 | Product 1 | Warehouse 1 | 20 | 20 |
| | | Warehouse 2 | 40 | 40 |
| | Product 2 | Warehouse 1 | 100 | 90 |
| | | Warehouse 2 | 60 | 50 |
| Problem 2 | Product 1 | Warehouse 1 | 20 | 20 |
| | | Warehouse 2 | 20 | 20 |
| | Product 2 | Warehouse 1 | 80 | 40 |
| | | Warehouse 2 | 40 | 40 |

**TABLE 1:** warehouse demand of each product.

Table II represents the results of the assignment problem model. The objective function aims at optimizing the number of vehicles to use. In particular, this table specifies the site of each vehicle as well as the vehicles which will be in maintenance for each day.

| | | Vehicle type | t 1 | t 2 | CPU |
|---|---|---|---|---|---|
| Problem 1 | depot 1 | Type1<br>Type2 | 13<br>5_6_8_9_10 | 13<br>5_6_7_8_9 | 0.02 |
| | Depot 2 | Type1<br>Type2 | 12_11<br>2_3_4 | 11_12<br>1_2_3 | |
| | Maint-enance | Type1<br>Type2 | 14<br>1_7 | 4_10 | |
| Problem 2 | Depot 1 | Type1<br>Type2 | 12<br>2_4_5_ 6 | 13<br>6_7 | 0.11 |
| | Depot 2 | Type1<br>Type2 | 11<br>1_3 | 11<br>3_5 | |
| | Maint-enance | Type1<br>Type2 | 14<br>7_8 | 12<br>1_2_4_9_10 | |

**TABLE 2:** Results obtained by Cplex

Since these solutions imply associations of various combinations, the number of routes in each one of solutions can differ. Thus, it is noted that the number of vehicles used can easily pass from fourteen to twelve in the first day and eleven in the second day.
Then, the number of trucks necessary is generally lower than the total number available vehicles, which is explained by the high cost of adding new trucks. Consequently, by preserving the number of vehicles obtained, it can be taken for asset that this number of trucks will be used as being the number of vehicles available in each warehouse. In this way, in the best of the cases, the model will tend to reduce this number.

| | **Optimal solution t1** | **Optimal solution t2** | **CPU t1** | **t2** | **Logistical solution t1** | **Logistical solution t2** |
|---|---|---|---|---|---|---|
| Problem 1 | Route2 : 11_3_11<br>Route3 : 11_4_11<br>Route4 : 11_7_11<br>Route5 : 0_9_0<br>Route6 : 0_1_0<br>Route 8: 0_8_2_0<br>Route9 : 0_5_0<br>Route10 : 0_6_0<br>Route11 : 11_5_8_7_11<br>Route12 : 11_2_9_3_11<br>Route13 :0_4_6_0<br>L10_1<br>L10_2 | Route1 : 11_1_11<br>Route2 : 11_4_11<br>Route3 : 11_2_11<br>Route5 : 0_8_10_3_0<br>Route6 : 0_6_0<br>Route7 : 0_7_0<br>Route8 : 0_9_0<br>Route11 : 11_6_1_11<br>Route12 : 11_5_4_11<br>Route13 :0_9_3_7_0<br>L2_1<br>L10_1 | 6229, 77sec | 18 280,03sec | Route2 : 11_3_11<br>Route3 : 11_4_11<br>Route4 : 11_7_11<br>Route5 : 0_6_2_0<br>Route6 : 0_5_0<br>Route8 : 0_2_8_0<br>Route9 : 0_1_0<br>Route10 : 0_9_0<br>Route11 : 11_3_4_7_11<br>Route12 : 11_6_5_2_11<br>Route13 :0_8_9_0<br>L10_1<br>L10_2 | Route1 : 11_4_11<br>Route2 : 11_3_8_11<br>Route3 : 11_7_11<br>Route5 : 0_10_0<br>Route6 : 0_2_0<br>Route7 : 0_9_0<br>Route8 : 0_6_0<br>Route11 : 11_2_1_7_11<br>Route12 : 11_4_6_11<br>Route13 :0_5_9_0<br>L10_1 |
| Total cost | 777 | 597 | | | 817 | 660 |
| Problem 2 | Route1 : 8_3_8<br>Route2 : 0_5_0<br>Route3 : 8_6_8<br>Route4 : 0_2_0<br>Route5 : 0_4_0<br>Route6 : 0_1_0<br>Route11 : 8_4_5_7_8<br>Route12 : 0_2_1_0<br>L3_1<br>L7_2 | Route3 : 7_4_7<br>Route5 : 7_5_7<br>Route6 : 0_2_0<br>Route7 : 0_3_6_0<br>Route11 :7_2_5_7<br>Route13 :0_6_4_1_0<br>L3_1<br>L1_2 | 0.84sec | 0.27 sec | Route1 : 8_3_8<br>Route2 : 0_5_0<br>Route3 : 8_4_8<br>Route4 : 0_2_0<br>Route5 : 0_6_0<br>Route6 : 0_1_0<br>Route11 : 8_2_3_8<br>Route12 : 0_4_5_1_0<br>L7_1<br>L7_2 | Route3 : 7_6_3_7<br>Route5 : 7_2_7<br>Route6 : 0_5_0<br>Route7 : 0_4_0<br>Route11 :7_2_5_7<br>Route13 :0_6_4_1_0<br>L3_1<br>L1_2 |
| Total cost | 594 | 487 | | | 663 | 498 |

**TABLE 3:** Optimal routes

In table 3, we present the results obtained for 15 customers and 11 vehicles as well as the optimal solution obtained by the exact method. The results obtained are compared with those obtained by the distributer. We notice that the manual method generates 40d more in problem 1 at day 1, 63d more in problem 1 at day 2, 69d more in problem 1 at day 1, 11d more in problem 1 at day 2. Thus, it is clear that the cost of routes differs between the solution obtained by the distributer method and that produced by the exact method. The error of the second method is probably due to a decision making during the distributions which eliminate certain possibilities. Consequently, even if the solution obtained is excellent, the optimal solution was impossible to reach.

## 6. CONCLUSION

The delivery of goods from a warehouse to a customer is an important and very practical problem of supply chain management. In this paper, we developed two mathematical models adapted to the constraints of the company. Some results obtained by Cplex are presented. As for further research, a wide range of test problems should be performed. It would be interesting to solve this problem by intelligent optimization or heuristics techniques.

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## Appendix A:

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Capacity** | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| **Fixed Cost at depot 1** | 40 | 40 | 40 | 40 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| **Fixed Cost at depot 2** | 50 | 50 | 50 | 50 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |

**Details of problem 1**

| **T1** | | | | | | **T2** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *N* | *X* | *Y* | *q* *Type1* | *q* *Type2* | *L* | *N* | *X* | *Y* | *q* *Type1* | *q* *TYpe2* | *L* |
| 0 | 35 | 35 | - | - | - | 0 | 35 | 35 | | | |
| 1 | 55 | 20 | 0 | 12 | 135 | 1 | 55 | 20 | 5 | 10 | 135 |
| 2 | 55 | 45 | 5 | 10 | 120 | 2 | 40 | 30 | 10 | 18 | 45 |
| 3 | 35 | 17 | 7 | 20 | 100 | 3 | 20 | 35 | 8 | 6 | 90 |
| 4 | 45 | 20 | 10 | 11 | 100 | 4 | 60 | 15 | 8 | 20 | 142 |
| 5 | 15 | 30 | 5 | 20 | 120 | 5 | 65 | 35 | 9 | 9 | 140 |
| 6 | 25 | 45 | 9 | 20 | 90 | 6 | 60 | 30 | 12 | 11 | 135 |
| 7 | 20 | 19 | 3 | 19 | 120 | 7 | 20 | 30 | 5 | 19 | 90 |
| 8 | 10 | 30 | 12 | 10 | 138 | 8 | 35 | 40 | 0 | 4 | 40 |
| 9 | 65 | 15 | 8 | 18 | 150 | 9 | 35 | 50 | 7 | 20 | 90 |
| 10 | 35 | 50 | 13 | 20 | 90 | 10 | 25 | 40 | 12 | 10 | 85 |
| 11 | 35 | 30 | - | - | - | 11 | 35 | 30 | | | |

| $V_{kt}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **T1** | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| **T2** | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

**Details of problem 2**

| **T1** | | | | | | **T2** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *N* | *X* | *Y* | *q* *Type1* | *q* *Type2* | *L* | *N* | *X* | *Y* | *q* *Type1* | *q* *TYpe2* | *L* |
| 0 | 35 | 35 | - | - | - | 0 | 35 | 35 | - | - | |
| 1 | 55 | 20 | 10 | 20 | 135 | 1 | 45 | 30 | 4 | 14 | 85 |
| 2 | 45 | 30 | 8 | 20 | 85 | 2 | 15 | 10 | 8 | 17 | 140 |
| 3 | 15 | 10 | 12 | 20 | 140 | 3 | 20 | 28 | 10 | 12 | 100 |
| 4 | 20 | 28 | 5 | 20 | 100 | 4 | 55 | 20 | 5 | 18 | 135 |
| 5 | 20 | 25 | 4 | 20 | 100 | 5 | 30 | 15 | 12 | 19 | 120 |
| 6 | 30 | 15 | 0 | 20 | 120 | 6 | 30 | 20 | 10 | 5 | 90 |
| 7 | 30 | 20 | 10 | 15 | 90 | 7 | 35 | 30 | - | - | |
| 8 | 35 | 30 | - | - | - | | | | | | |

| $V_{kt}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **T1** | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| **T2** | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |