

## Two Phase Algorithm for Solving VRPTW Problem

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### Abstract

Vehicle Routing Problem with Time Windows (VRPTW) is a well known NP hard combinatorial scheduling optimization problem in which minimum number of routes have to be determined to serve all the customers within their specified time windows. Different analytic and heuristic approaches have been tried to solve such problems. In this paper we propose a two phase method which utilizes Genetic algorithms as well as random search incorporating simulated annealing concepts to solve VRPTW problem in various scenarios.

**Keywords:** Vehicle Routing Problem with Time Windows, Genetic Algorithm, Random Search Algorithm, Simulated Annealing

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### 1. INTRODUCTION

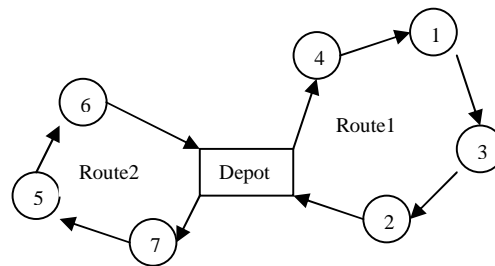
The Vehicle Routing Problem (VRP) is a well known NP-hard combinatorial optimization problem. It arises in distribution systems that usually involve scheduling in constrained environment. It lies at the heart of distribution management. It is faced each day by companies and organizations engaged in the delivery and collection of goods and people. Much progress has been accomplished since the publication of the first article on the "truck dispatching" problem [23]. Several variants of the basic problem have been proposed and different formulations developed. Exact solutions techniques as well as numerous heuristics have also been developed and tried for solving the vehicle routing problems.

In VRP a fleet of vehicles must visit a number of geographically scattered customers. All vehicles start and end at a common home base, called the depot. Each customer must be visited exactly once. The cost of traveling between each pair of customers and between the depot and each customer is given. Additionally each customer demands a certain quantity of goods delivered (known as customer demand). Each vehicle has an upper limit on the amount of goods that it can carry (known as the capacity). All vehicles are of the same type and hence have the same capacity. The problem is for a given scenario to plan routes for each vehicle, such that all routes start and end at the depot, each customer is served by exactly one vehicle, the total demand at locations enroute do not exceed the carrying capacity of the vehicle in a way that the overall cost of the routes are minimized. The Vehicle Routing Problem with Time Windows (VRPTW) is a generalization of VRP. VRPTW is one of its variant amongst Capacitated VRP, Multiple Depot VRP, VRP with Backhauls and VRP with Pick-up and Deliveries.

VRPTW is VRP with additional restriction that each customer has a time window. The vehicle can visit the customer in this specified time window only. It does not accept a vehicle after the latest time specified in the time window. However, if a vehicle arrives at such a destination prior to earliest specified time of the window, vehicle must wait and service will not start until the time window of the customer actually opens. Figure 1 shows a graphical model of VRPTW and its solution.

The objective of VRPTW is to service all the customers as per their requirement while minimizing the number of vehicles required as well as the total travel distance by all the vehicles used without violating capacity constraints of the vehicles and the customers' time window requirements such that each customer is visited only once by one of the vehicles. All the routes are to start and ultimately end at the originating depot.

VRPTW arise in a variety of real life situations. VRPTW arises in milk float, mail delivery, school bus routing, solid waste collection, heating oil distribution, newspaper distribution, transportation of persons, parcel pick-up and delivery, dial-a-ride systems, airline/railway fleet routing and several such other situations.



**FIGURE 1:** Typical output for VRPTW.

VRPTW is NP-hard. It has been extensively investigated in recent years using analytic optimization techniques, heuristics and meta-heuristics approaches. The term heuristics and meta-heuristics are usually associated with Random Search based techniques, Simulated Annealing based techniques, Tabu Search, Genetic Algorithms, Particle Swarm Optimization, Ant Colony Optimization, cross-entropy, stochastic approximation, multi-start clustering algorithms.

The available literature on VRPTW can be broadly divided into two categories: exact optimization and heuristic algorithms. Using exact optimization techniques, [10], [4], [11], [12] obtained significant improvements in Solomon's benchmark problem. Survey of the VRPTW literature by heuristics has been given by [3], [14]. Tabu search is used by [5], [25] to solve these problems whereas [7], [26], [27] considered ant colony optimization approach. Large neighborhood search (LNS) is applied by [18]. This was extended by [17] as Adaptive-LNS approach to solve VRPTW problems. Parallelization of a two-phase metaheuristic technique proposed by [9] for solving VRPTW. A complete survey of the VRPTW literature, which includes methods of both the categories, is given by [6].

The Genetic Algorithm (GA) approach was proposed by [8] in 1975. It is an adaptive heuristic search method that mimics evolution through natural selection. It works by combining selection, crossover and mutation operations of genes. Genetic Algorithm and random search heuristics have been frequently used for solving combinatorial optimization problems which are nondeterministic polynomial hard or nondeterministic polynomial complete. Whereas genetic algorithms search for the optimal solution in a large region of search space making use of genetic operators, random search based techniques usually achieve this in conjunction with annealing approach.

GA approach was first used by Blanton et al. [2] to solve VRPTW. They hybridized GA approach with a greedy heuristic. A cluster-first, route-second method using genetic and local search optimization was used by [22] and GENEROUS by [15]. A multi-objective representation of VRPTW using pareto-ranking was used by [13]. [1], [21] and many more also used GA for solving VRPTW problems.

Genetic Algorithms have been frequently used for solving VRPTW problem which is NP-hard. However it has been noticed that the techniques do not guarantee a near optimal solution in each case. In some cases the yielded solution is far from the global optimal. Keeping this in view an attempt has been made in the present study to first search for the optimal solution of the VRPTW problem using conventional genetic algorithm approach and once the solution is achieved to perturb it randomly in annealing type manner to see if a still better solution can be achieved.

The present study is organized as under: In section 2, we describe the proposed two phase optimization technique for solving VRPTW. Section 3 describes the details of genetic algorithm approach adopted by us for solving VRPTW. Use of random search approach in annealing type manner for further improvement of the obtained results is next explained in section 4. Application of the technique on selected 30 problems taken from [19] benchmark set of problems is considered in section 5. Conclusions based on present study are finally drawn in section 6.

## **2. PROPOSED ALGORITHM FOR VRPTW**

The proposed algorithm for solving vehicle routing problem with time windows works in two phases. In the first phase it uses a genetic search based algorithm described in section 3 to generate an optimal solution. In the next phase it perturbs iteratively the optimal solution obtained in Phase I in a random search manner incorporating annealing concept to see if a still better solution is possible or not. The proposed algorithm works as under:

### **Phase I: Genetic Algorithm phase**

- i) Build an initial population and evaluate fitness value of each individual in the population.
- ii) Find the individual which has best fitness value.
- iii) Generate new individuals by applying genetic operators to suitably selected members from the current population.
- iv) Evaluate fitness of new individuals and place them in new population.
- v) Repeat step (ii) to (iv) till specified stopping criterion satisfied.
- vi) Store the individual with the best fitness value.

### **Phase II: Random Search Phase**

- i) Use the best solution obtained in phase I as current solution.
- ii) Randomly perturb the current solution to obtain a new solution and evaluate the fitness.
- iii) Is new solution better than the current one? If yes, replace current solution by the new one else replace current solution by the new one in annealing type manner.
- iv) Repeat step (ii) and (iii) iteratively a specified number of times.

The final solution achieved in phase II is the desired optimal solution. A special feature of the proposed two phase algorithm is that the Phase II does not start random neighbourhood search from an arbitrarily chosen solution but with the best solution achieved in Phase I using genetic algorithm approach. So there are great chances of the obtained solution being better than the one provided by genetic algorithm approach in case such a solution exists.

## **3. GENETIC ALGORITHM PHASE**

In this phase an initial population is first generated. After building the initial population, all individuals are evaluated according to the fitness criteria. The evolution continues with

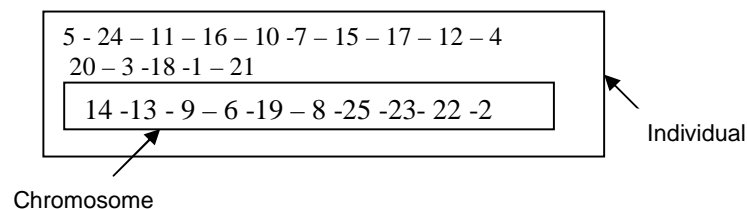
tournament selection in which good individuals are selected for reproduction. In each generation two best individuals are preserved for the next generation without being subjected to genetic operations. Crossover and mutation operations are then applied to modify the suitably selected individuals to form new feasible individuals for the population. The detailed description of genetic algorithm used to solve vehicle routing problem with time windows is as under.

### 3.1 Chromosome & Individual Representation

The chromosomes in genetic algorithms are often represented as a fixed-structure bit string, for which the bit positions are assumed to be independent and context insensitive. However, such a representation is not suitable for VRPTW, which is an order-oriented NP-hard optimization problem where specific sequences among customers are essential.

For solving VRPTW, the representation that we have used is same as used by [24]. The representation is as follows: Each customer is assigned a unique integer identifier  $i$ , where  $i \in N$ . An individual, which is a collection of chromosomes, represents a complete routing solution. Depending on how the customers are routed and distributed, every individual can have different number of routes for the same data set.

Each chromosome represents a route, which is variable in length. It contains a sequence of customers in the order in which they are visited by the vehicle. A different vehicle is needed for every chromosome of the individual. Every individual and every route ensure to be feasible, in terms of capacity and time window constraints. The central depot is not considered in this representation, because all routes necessarily start and end at the depot. However time from the depot to first customer of the route and time from last customer of the route to depot is taken into account. Figure 2 represents an illustrative complete routing solution for a problem instance with 25 customers, consisting of 3 routes which are served by 3 vehicles; genetically we call each route a chromosomes and complete solution an individual.



**FIGURE 2:** An individual with 3 chromosomes (vehicles / routes). It represents a solution for a problem instance with 25 customers.

### 3.2 Initial Population

An initial population is built such that each individual (which represents a complete routing solution in our case) is a feasible solution. In other words every individual and every chromosome/route in the selected population satisfies time window and capacity constraints. The first feasible solution is generated using Push Forward Insertion Heuristics (PFIH) introduced by [19]. This method has been frequently used in literature. Details of this method are available in [22]. Rest of the solutions of initial population are generated by selecting the customers in a random manner and inserting them in an existing route, if such a possibility exists, otherwise a new route is created. Any customer that violates any constraint is deleted and a new route is added to serve such a customer. This process is repeated until all the customers get served and a feasible initial population has been generated.

### 3.3 Fitness

The fitness function measures the quality of the represented solutions. As soon as all the individuals have been created, they are ranked as per their fitness. It is commonly taken as the

objective function of the optimization problem but it may not necessarily coincide with it. For solving VRPTW using the proposed algorithm following fitness criteria's have been considered:

1. *Inverted Distance (INVD)*: Inverse of the total traveled distance is used to calculate the fitness of the individuals.
2. *Distance Traveled and Number of Routes (DR)*: In this case we minimize the total distance traveled keeping at the same time the number of vehicles as low as possible as each route requires one vehicle to operate it.
3. *Number of Routes and Distance traveled (RD)*: Same as (2) however with the priorities interchanged. First priority is to reduce the number of routes and second priority is to reduce total distance traveled.
4. *Weighted Sum Method (WM)*: In this case effort has been made to minimize weighted sum of two objectives. For VRPTW, the weighted sum objectives which has been minimized is:

$$F(x) = \alpha \cdot V + \beta \cdot TD$$

where  $\alpha$  and  $\beta$  are suitable weight coefficients associated with total number of vehicles,  $V$  and total distance traveled,  $TD$  by vehicles. The weight values of the coefficients used have to be established empirically (we have used  $\alpha = 100$  and  $\beta = 0.001$ ) for this study.

### 3.4 Selection and Elitism

In selection, parents are selected for crossover. There are many methods proposed in the literature for this. In this study, an  $x$ -way tournament selection procedure has been used. Here  $x$  individuals are randomly selected and then the individual with highest fitness is declared the winner. This process is repeated until the number of selected individuals equals the number necessary for crossover. In this study, tournament size, i.e.  $x$  has been taken to be 3.

In the elitism process the good individuals are retained for reproduction. This ensures that the best solution obtained from the present population is copied unaltered in the next population. We replace the two worst individuals in the new population with the best two individuals of the parent population.

### 3.5 Crossover and Mutation

The classical single/double point crossover operators are relevant to string entries that are order less. They put two integer/binary strings side by side and make a cut point (or two cut points) on both of them. A crossover is then completed by swapping the portions after the cut point (or between the cut points) in both the strings. However it is not appropriate for scheduling problems like TSP or VRP where sequence or order among the integers is very important because duplication and omission of vertices can produce infeasible sequences in the offspring. Therefore in present study Route-Exchange crossover is used. Earlier this crossover has been applied by [21], [13]. However they choose the best route according to the objectives for crossover. In the present study, once a pair of individuals is selected for crossover, efforts are made to exchange a route that has minimum number of nodes in each of the two individuals. To ensure that all individuals are feasible routing solutions after crossover any duplication is deleted.

Mutation is necessary for inserting new characteristics that are not present in the current individuals. Without mutation the search gets limited to a very small area in the feasible region. In present study effort has been made to achieve this by transferring customers from a route that has minimum number of customers to other routes if feasible so that such a route gets deleted (if possible) and the number of routes reduced.

#### 4. RANDOM SEARCH PHASE

The application of random search concept incorporating simulated annealing in Phase II of the algorithm to solve the VRPTW is as follows: Initially the best solution, say  $S$  of the Phase I of the genetic algorithm is taken as the starting current solution. From this a new solution  $S'$  is obtained by randomly moving a customer of the current solution from one route to another route where it is feasible. The new solution if it decreases the objective function value (or leaves it unchanged) is accepted to replace the current solution; else it is accepted to replace the current solution in annealing type manner. More precisely, the new solution  $S'$  is accepted as the new current solution if  $\Delta E \leq 0$ , (where  $\Delta E = \text{Fitness}(S') - \text{Fitness}(S)$ ) else to ensure the search to escape a local optimum, solutions that increases the objective function value are accepted if

$$\exp(-\Delta E/T_k) > \theta \quad (4.1)$$

where  $\theta$  is a randomly selected number between  $[0, 1]$ , if  $\Delta E > 0$ , where  $T_k$  is a parameter called the "temperature". (The value of  $T_k$  is gradually decreased from a relatively large value to a small value close to zero. These values are controlled by a cooling schedule which specifies the initial and temperature values at each stage of the algorithm. Eqn. (4.1) implies that in a minimization problem large increases in objective function, so called uphill moves, are more likely to be accepted when  $T_k$  is high. As  $T_k$  approaches zero most uphill moves are rejected.

In order to achieve good optimization results the simulated annealing metaheuristic has to be adjusted specific to the problem. With respect to the VRPTW following parameters have been taken into account:

- a) *Initial temperature of annealing  $T_0$* : If  $T_0$  is too high then almost all new solutions are accepted and the search produces a series of random solutions. When  $T_0$  is too low, very few movements are allowed which reduce the scope of the search. After a series of experiments we decided to set constant initial temperature  $T_0=100$ .
- b) *Cooling schedule*: One of the most popular temperature reduction functions is by [28]. It is based on geometric reduction  $T_{k+1} = T_k * \gamma$ . In the present work we have set parameter  $\gamma$  as 0.96.
- c) *Number of annealing steps executed in each temperature*: It is usually related to the size of a solution neighbourhood. In our computational experiments the numbers of annealing steps are set as  $n^2$ , where  $n$  is the number of customers in the problem considered.
- d) *Termination condition*: We set the termination criteria as  $T_0=0$ .

Contrary to the classical approach in which a solution to the problem is taken as the last solution obtained in the annealing process, we memorize the best solution found during the whole annealing process and record it as the best solution found. The algorithm has been coded in C++ and run on an Intel(R) Core(TM) 2 Duo 2.0 GHz for solving chosen test problems.

#### 5. NUMERICAL RESULTS

In this section we present our computational experience of using the proposed algorithm in solving a set of benchmark test problems selected from Solomon's [19] set of problems.

In the present study 30 problems have been chosen from Solomon's set of problems. Of these 15 problems are of 25 customers, 10 problems of 50 customers and 5 problems of 100 customers. Each problem was solved ten times using developed algorithm. The best and the worst results obtained in each problem in ten trials are listed in the tables. We first applied the two phase algorithm considering the vehicle routing problem as a single objective problem, thus applied the fitness criteria INVD. Results obtained in phase I and their refinement after the phase II is shown in TABLE 1 for 25 customer problems, TABLE 2 for 50 customer problems and TABLE 3 for 100

customer problems. Out of ten solutions obtained in each case the Best solution and Worst solution of each problem after completion of phase I and phase II are listed in these tables. Stopping criterion used for phase I is a maximum of 1000 generations or when no improvement is observed in the objective function value of the best individual in consecutive 100 generations. In the Phase I, the population size was taken as 100; crossover rate was taken to be 0.80 and mutation rate 0.20.

However VRPTW is in reality a multi objective problem in which we went to minimize the number of routes used as well as the total distance traveled. Thus we have used the same two phase algorithm to solve VRPTW considering it as a multi-objective problem in order to check if performance in both objectives can be simultaneously improved. The performance of proposed algorithm is tested on the same set of 30 benchmark problems. TABLE 4, 5 and 6 shows the results obtained after phase II for the three multiobjective criteria's DR, RD and WM for 25 customer, 50 customer and 100 customer problems respectively.

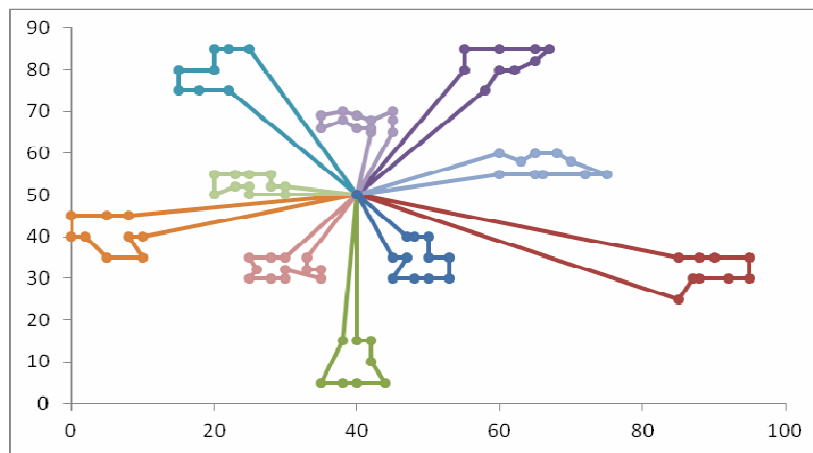
Tables also present a summary of the results obtained and their comparison with the best-known solutions available to us from literature. Bold numbers in tables indicate that the obtained solution are same as the best-known or has yielded an improvement on the currently best known solution in the literature.

### 5.1 Discussion on the Results

Results presented in the TABLE 1, 2 and 3 shows the results after completion of genetic algorithm Phase I and random search based Phase II. For 25 customer problems there are only four cases out of fifteen in which no further refinement in the solutions is seen after Phase I. For 50 customer problems there are two such cases out of ten and for 100 customer problems there is one such case out of five. In all out of thirty problems, in twenty three cases Phase II refines solution obtained after Phase I.

For 25 customer problem the proposed algorithm has produced new improved results in six problems out of fifteen problems investigated as shown in TABLE 1. The obtained results require less number of vehicles, however at the expense of additional distance to be traveled. In TABLE 2 for 50 customer problems; five problems from set R2 show improvement while one problem from each C1 and C2 set have identical solution with the best-known solutions. In R201 problem obtained results reflect significant decrease in the requirement of number of vehicles, (maximum reduction is two and minimum is one) than the number of vehicles used in best-known solutions. In case of 100 customer problems proposed algorithms provide improved results in one problem out of five considered as shown in TABLE 3 while another problem provide solution identical with the best-known solution available in the literature. In this case the improvement is in terms of less total distance traveled than the best-known solution. However, it needs one extra vehicle than the number of vehicles used in the best-known. In the remaining problems solutions obtained are either identical to the best known solutions or within 10% of the best-known solution. Figure 3 shows the geographical representation of solution obtained for problem C101 for 100 customers.

TABLE 4, 5 and 6 shows the results obtained after phase II for the three multiobjective criteria's DR, RD and WM for 25 customer, 50 customer and 100 customer problems respectively. In TABLE 4 for 25 customer problems the proposed algorithm has yielded either improved results in terms of lesser number of vehicles required than the best known solutions or identical solutions with the best-known solutions in ten problems out of selected fifteen problems. In some cases such as RC203 and RC206 the improvement is quite significant as it decreases the need of number of vehicles by one to two vehicles compared to the number of vehicles used in the best-known solutions. However this improvement in terms of lesser number of vehicles required is at cost of more distance to be traveled. In case of remaining five problems obtained solutions are within 10% of the best-known solutions. A comparison of the results obtained with three objectives shows that objectives DR and WM provide better results than objective RD.



**FIGURE 3:** Geographical representation of solution obtained for problem C101 for 100 customers.

In case of 50 customer problems for multiobjective criteria's DR, RD and WM results obtained are presented in Table 5. The use of the two phase algorithm has provided six improved solutions with all the three multi-objective criteria's. Also DR criterion has produced solutions identical with the best-known in two cases. Thus in all out of ten selected problems, eight problems either yielded improved results in terms of lesser number of vehicles required than the best known solutions. However this improvement is at the expense of extra distance to be traveled. (Maximum reduction in the number of vehicles required is three and minimum one.) In remaining two problems solutions are within 10% of the best-known solution. Performance wise the three objectives may be ranked as WM, DR and RD, in that order. In the case of 100 customer problems for multiobjective criteria's results are shown in TABLE 6; proposed algorithms produced improved results in one problem out of five considered while one problem yielded identical solution as the best-known solution available in literature. In the present case the improvement is in terms of less total distance traveled than the best-known solution. However, it needs one more vehicle than the number of vehicles used in the best-known. In remaining three problems solutions are within 10% of the best-known solution. Performance wise WM objective is again superior to the remaining two. Figure 4 shows the geographical representation of solution obtained for problem R105 for 100 customers.

Lesser number of vehicles means saving in terms of needed man power requirement and vehicle maintenance cost. It also means less chaos at the depot. However, solutions which reported lesser number of vehicles reported more distance to travel, adding to fuel cost. However fuel cost is comparatively less expensive than arranging additional vehicles and man power needed to operate them. Moreover, it is easy for the management to handle the things with less number of vehicles.

## 6. CONCLUSIONS

Vehicle routing problem with time windows involves the optimization of routes for multiple vehicles so as to meet all constraints and to minimize the number of vehicles needed and total distance traveled. The proposed two phase algorithm is based on genetic algorithm and random search incorporating simulated annealing concept. Performance of proposed algorithms is comparable to those available in literature and in some cases even better in terms of number of vehicles which means less fuel, manpower and vehicle maintenance cost with more distance to travel. We solved each problem ten times and have presented the best and worst achieved solutions to indicate the range of variations in the solution obtained. The variation of the best solution from worst solution is generally in the range of 20 to 40 percent indicating thereby that it is advisable to solve a problem more than once to achieve the best results. This is due to the probabilistic nature of the techniques which search a near optimal solution. As for future work, it may be interesting to test proposed algorithm on some application of VRPTW.



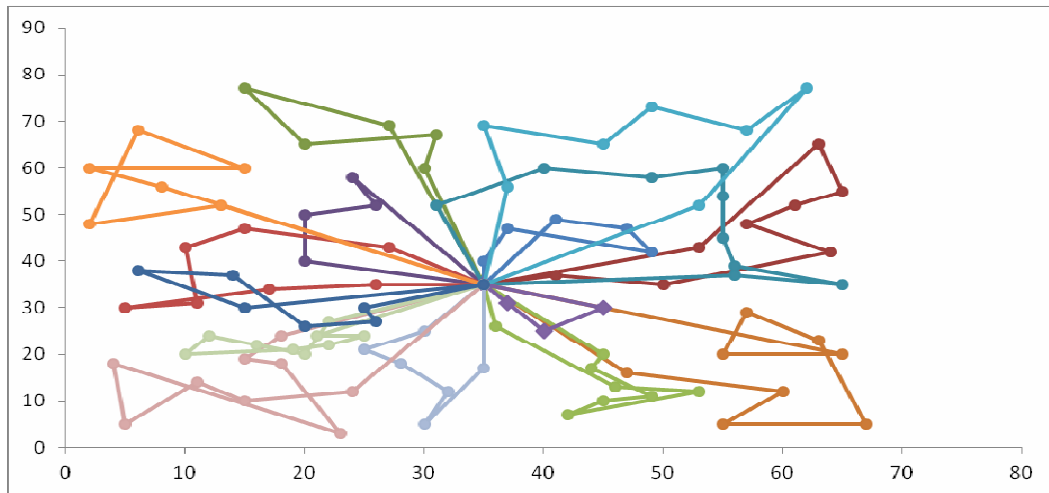


FIGURE 4: Geographical representation of solution obtained for problem R105 for 100 customers.

Problem	INVD				Best Known [Ref.] (No. of Vehicles/ distance traveled)
	After Genetic Algorithm Phase		After Refining the best obtained from GA with SA in Phase II		
	Best (No. of Vehicles/ distance traveled)	Worst (No. of Vehicles/ distance traveled)	Best (No. of Vehicles/ distance traveled)	Worst (No. of Vehicles/ distance traveled)	
C201	2/ 237.15	2/ 282.25	2/ <b>214.7</b>	2/ 250.56	<b>2/ 214.7</b> [11]
R101	8/ 625.03	9/ 661.34	8/ 623.52	9/ 645.20	<b>8/ 617.1</b> [11]
R102	7/ 591.51	7/ 628.93	7/ 586.05	7/ 610.46	<b>7/ 547.1</b> [11]
R105	6/ 569.70	6/ 587.76	<b>5/ 559.84</b>	6/ 575.00	<b>6/ 530.5</b> [11]
R109	5/ 522.05	5/ 558.03	5/ 480.23	5/ 554.98	<b>5/ 441.3</b> [11]
RC105	4/ 412.38	4/ 506.66	4/ 412.38	4/ 495.34	<b>4/ 411.3</b> [11]
RC106	3/ 364.97	4/ 459.10	3/ 364.97	4/ 431.43	<b>3/ 345.5</b> [11]
RC201	3/ 385.81	3/ 454.50	3/ 385.81	3/ 452.94	<b>3/ 360.2</b> [12]

RC202	3/ 361.64	3/ 465.04	3/ 361.64	3/ 433.73	3/ <b>338.0</b> [10]
RC203	2/ 468.98	3/ 482.96	2/ <b>411.47</b>	3/ 441.22	3/ <b>326.9</b> [4]
RC204	2/ 438.45	2/ 475.28	2/ <b>401.05</b>	2/ 452.17	3/ <b>299.7</b> [4]
RC205	3/ 399.10	3/ 482.34	3/ 363.46	3/ 432.66	3/ <b>338.0</b> [12]
RC206	2/ 526.33	2/ 548.06	2/ <b>503.28</b>	2/ 528.46	3/ <b>324.0</b> [10]
RC207	2/ 477.63	2/ 562.91	2/ <b>433.23</b>	2/ 533.65	3/ <b>298.3</b> [10]
RC208	1/ 440.80	1/ 500.75	1/ <b>427.26</b>	1/ 483.79	2/ <b>269.1</b> [4]

**TABLE 1:** Results generated by Two Phase Algorithm for Solomon's 25 customers set Problems (considering VRPTW as single-objective optimization problem).

Problem	INVD				Best Known [Ref.]  (No. of Vehicles/ distance traveled)
	After Genetic Algorithm Phase		After Refining the best obtained from GA with SA in Phase II		
	<b>Best</b> (No. of Vehicles/ distance traveled)	<b>Worst</b> (No. of Vehicles/ distance traveled)	<b>Best</b> (No. of Vehicles/ distance traveled)	<b>Worst</b> (No. of Vehicles/ distance traveled)	
C101	5/ 385.90	6/ 444.27	5/ <b>362.5</b>	6/ 411.30	5/ <b>362.5</b> [11]
C201	3/ 408.73	3/ 390.26	3/ 390.26	3/ 434.72	3/ <b>360.2</b> [12]
C205	3/ 389.83	3/ 459.36	3/ <b>360.2</b>	3/ 435.24	3/ <b>360.2</b> [12]
R101	12/ 1097.97	12/ 1190.42	12/ 1055.56	12/ 1178.27	12/ <b>1044</b> [11]
R201	5/ 1108.55	5/ 1262.96	5/ <b>1099.81</b>	5/ 1206.59	6/ <b>791.9</b> [10]
R202	4/ 1127.16	4/ 1150.84	4/ <b>1059.12</b>	4/ 1149.09	5/ <b>698.5</b> [10]
R203	4/ 1097.14	4/ 1172.92	4/ <b>1047.22</b>	4/ 1125.25	5/ <b>605.3</b> [4]

R206	3/ 1023.04	3/ 1179.46	<b>3/ 949.08</b>	3/ 1167.75	<b>4/ 632.4 [4]</b>
R209	3/ 1165.67	3/ 1208.34	<b>3/ 1145.18</b>	3/ 1198.22	4/ 600.6 [10]
RC101	8/ 981.96	9/ 1024.66	8/ 981.96	9/ 1056.65	<b>8/ 944 [11]</b>

**TABLE 2:** Results generated by Two Phase Algorithm for Solomon's 50 customers set Problems (considering VRPTW as single-objective optimization problem).

Problem	INVD				Best Known [Ref.] (No. of Vehicles/ distance traveled)
	After Genetic Algorithm Phase		After Refining the best obtained from GA with SA in Phase II		
	<i>Best</i> (No. of Vehicles/ distance traveled)	<i>Worst</i> (No. of Vehicles/ distance traveled)	<i>Best</i> (No. of Vehicles/ distance traveled)	<i>Worst</i> (No. of Vehicles/ distance traveled)	
C101	10/ 838.07	11/ 1054.11	<b>10/ 828.94</b>	11/ 1022.68	<b>10/ 828.94 [16]</b>
R101	21/ 1788.7	21/ 1877.92	20/ 1736.60	21/ 1869.44	<b>19/ 1650.8 [16]</b>
R102	18/ 1516.67	19/ 1664.11	18/ 1516.67	19/ 1661.76	<b>17/ 1486.12 [16]</b>
R105	15/ 1391.85	17/ 1546.56	15/ 1380.05	17/ 1539.5	<b>14/ 1377.11 [18]</b>
RC101	15/ 1775.79	18/ 1960.92	<b>15/ 1640.98</b>	18/ 1953.77	<b>14/ 1696.94 [20]</b>

**TABLE 3:** Results generated by Two Phase Algorithm for Solomon's 100 customers set Problems (considering VRPTW as single-objective optimization problem).

Problem	DR		RD		WM		Best Known [Ref.]
	<i>Best</i> (No. of Vehicles/ distance traveled)	<i>Worst</i> (No. of Vehicles/ distance traveled)	<i>Best</i> (No. of Vehicles/ distance traveled)	<i>Worst</i> (No. of Vehicles/ distance traveled)	<i>Best</i> (No. of Vehicles/ distance traveled)	<i>Worst</i> (No. of Vehicles/ distance traveled)	
C201	<b>2/ 214.7</b>	2/ 250.56	<b>2/ 214.7</b>	2/ 250.56	<b>2/ 214.7</b>	2/ 282.25	<b>2/ 214.7 [11]</b>

R101	8/ 618.33	8/ 665.83	8/ 623.52	8/ 677.80	8/ 629.95	8/ 651.44	<b>8/ 617.1</b> [11]
R102	7/ 586.00	7/ 620.21	7/ 590.41	7/ 668.86	7/ 584.20	7/ 628.93	<b>7/ 547.1</b> [11]
R105	<b>5/</b> <b>588.57</b>	6/ 628.72	<b>5/</b> <b>591.89</b>	6/ 609.90	<b>5/</b> <b>556.72</b>	6/ 612.57	<b>6/ 530.5</b> [11]
R109	5/ 480.09	5/ 575.80	5/ 483.94	5/ 557.77	5/ 461.71	5/ 566.09	<b>5/ 441.3</b> [11]
RC105	4/ 419.72	4/ 508.64	4/ 421.73	4/ 507.84	4/ 419.72	4/ 506.66	<b>4/ 411.3</b> [11]
RC106	3/ 363.82	4/ 443.43	3/ 371.21	4/ 455.30	3/ 375.74	4/ 463.63	<b>3/ 345.5</b> [11]
RC201	<b>2/</b> <b>519.35</b>	2/ 621.95	<b>2/</b> <b>519.35</b>	2/ 612.85	<b>2/</b> <b>496.37</b>	2/ 608.03	<b>3/ 360.2</b> [12]
RC202	<b>2/</b> <b>478.37</b>	2/ 560.42	<b>2/</b> <b>509.95</b>	2/ 597.46	<b>2/</b> <b>478.84</b>	2/ 554.53	<b>3/ 338.0</b> [10]
RC203	<b>2/</b> <b>419.02</b> <b>1/</b> <b>524.38</b>	2/ 568.73	<b>2/</b> <b>427.57</b> <b>1/</b> <b>524.38</b>	2/ 572.42	<b>2/</b> <b>441.76</b> <b>1/</b> <b>521.53</b>	2/ 525.48	<b>3/ 326.9</b> [4]
RC204	<b>2/</b> <b>382.41</b>	2/ 485.68	<b>2/</b> <b>391.66</b>	2/ 483.27	<b>2/</b> <b>386.53</b>	2/ 485.59	<b>3/ 299.7</b> [4]
RC205	<b>2/</b> <b>549.29</b>	2/ 697.92	<b>2/</b> <b>537.83</b>	2/ 681.46	<b>2/</b> <b>546.23</b>	2/ 653.75	<b>3/ 338.0</b> [12]
RC206	<b>2/</b> <b>497.02</b> <b>1/</b> <b>565.59</b>	1/ 648.04	<b>2/</b> <b>510.77</b> <b>1/</b> <b>566.86</b>	1/ 664.53	<b>2/</b> <b>483.93</b> <b>1/</b> <b>565.59</b>	1/ 653.26	<b>3/ 324.0</b> [10]
RC207	<b>2/</b> <b>433.76</b>	2/ 558.87	<b>2/</b> <b>435.32</b>	2/ 560.72	<b>2/</b> <b>424.39</b>	2/ 562.91	<b>3/ 298.3</b> [10]
RC208	<b>1/</b> <b>420.53</b>	1/ 483.79	<b>1/</b> <b>425.94</b>	1/ 500.75	<b>1/</b> <b>427.78</b>	1/ 504.51	<b>2/ 269.1</b> [4]

**TABLE 4:** Results generated by Two Phase Algorithm for Solomon's 25 customers set Problems (considering VRPTW as multi-objective optimization problem).

Problem	DR		RD		WM		Best Known [Ref.]
	<i>Best</i> (No. of Vehicles/ distance traveled)	<i>Worst</i> (No. of Vehicles/ distance traveled)	<i>Best</i> (No. of Vehicles/ distance traveled)	<i>Worst</i> (No. of Vehicles/ distance traveled)	<i>Best</i> (No. of Vehicles/ distance traveled)	<i>Worst</i> (No. of Vehicles/ distance traveled)	
C101	<b>5/ 362.5</b>	6/ 443.36	5/ 372.42	6/ 458.45	5/ 386.27	6/ 469.57	<b>5/ 362.5</b> [11]
C201	<b>2/ 501.13</b>	2/ 561.43	<b>2/ 546.73</b>	2/ 561.43	<b>2/ 501.13</b>	2/ 561.43	<b>3/ 360.2</b> [12]
C205	<b>3/ 360.2</b>	3/ 471.24	3/ 389.83	3/ 476.49	3/ 385.30	3/ 473.31	<b>3/ 360.2</b> [12]
R101	12/ 1080.71	12/ 1205.93	12/ 1093.91	12/ 1178.27	12/ 1088.71	12/ 1200.00	<b>12/ 1044</b> [11]
R201	3/ 1220.29	3/ 1337.42	3/ 1222.49	3/ 1344.47	3/ 1208.38	3/ 1338.00	<b>6/ 791.9</b> [10]
R202	<b>3/ 1054.11</b>	3/ 1231.76	<b>3/ 1057.32</b>	3/ 1252.27	<b>3/ 1041.92</b>	3/ 1205.38	<b>5/ 698.5</b> [10]
R203	<b>4/ 1025.96</b>	4/ 1146.74	<b>4/ 1039.82</b>	4/ 1173.38	<b>3/ 1233.18</b>	3/ 1336.97	<b>5/ 605.3</b> [4]
R206	<b>3/ 959.32</b>	3/ 1197.85	<b>3/ 964.95</b>	3/ 1138.22	<b>3/ 897.43</b>	3/ 1181.75	<b>4/ 632.4</b> [4]
R209	<b>2/ 1187.20</b>	3/ 1208.34	<b>2/ 1182.49</b>	3/ 1198.22	<b>2/ 1157.29</b>	3/ 1196.76	<b>4/ 600.6</b> [10]
RC101	8/ 982.62	10/ 1043.85	8/ 986.68	10/ 1037.84	8/ 977.51	10/ 1039.26	<b>8/ 944</b> [11]

**TABLE 5:** Results generated by Two Phase Algorithm for Solomon's 50 customers set Problems (considering VRPTW as multi-objective optimization problem).

Problem	DR		RD		WM		Best Known [Ref.]
	Best (No. of Vehicles/ distance traveled)	Worst (No. of Vehicles/ distance traveled)	Best (No. of Vehicles/ distance traveled)	Worst (No. of Vehicles/ distance traveled)	Best (No. of Vehicles/ distance traveled)	Worst (No. of Vehicles/ distance traveled)	
C101	10/ 829.70	11/ 1021.22	10/ 838.07	11/ 1035.71	<b>10/ 828.94</b>	12/ 1062.10	<b>10/ 828.94</b> [16]
R101	20/ 1751.74	21/ 1891.10	20/ 1760.08	21/ 1873.43	20/ 1740.18	21/ 1874.66	<b>19/ 1650.8</b> [16]
R102	18/ 1503.71	19/ 1723.94	18/ 1519.92	20/ 1747.98	18/ 1487.29	20/ 1718.42	<b>17/ 1486.12</b> [16]
R105	15/ 1381.23	17/ 1567.47	15/ 1391.85	17/ 1554.88	<b>15/ 1370.02</b>	17/ 1558.35	<b>14/ 1377.11</b> [18]
RC101	15/ 1723.18	18/ 1998.63	15/ 1738.55	18/ 1983.91	15/ 1717.31	18/ 1979.89	<b>14/ 1696.94</b> [20]

**TABLE 6:** Results generated by Two Phase Algorithm for Solomon’s 100 customers set Problems (considering VRPTW as multi-objective optimization problem).

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