Volume 1, Issue 2

Number of issues per year: 6

International Journal of Scientific and Statistical Computing (IJSSC)

ISSN : 2180-1339

INTERNATIONAL JOURNAL OF SCIENTIFIC AND STATISTICAL COMPUTING (IJSSC)

Volume 1, Issue 2, 2011

Edited By
Computer Science Journals
www.cscjournals.org

INTERNATIONAL JOURNAL OF SCIENTIFIC AND STATISTICAL COMPUTING (IJSSC)

Book: 2011 Volume 1, Issue 2
Publishing Date: 08-02-2011
Proceedings
ISSN (Online): 2180 -1339

IJSSC Journal is a part of CSC Publishers
http://www.cscjournals.org

Published in Malaysia

Typesetting: Camera-ready by author, data conversation by CSC Publishing Services – CSC Journals, Malaysia

CSC Publishers

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A Fuzzy Arithmetic Approach for Perishable Items in Discounted Entropic Order Quantity Model

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Abstract

This paper uses fuzzy arithmetic approach to the system cost for perishable items with instant deterioration for the discounted entropic order quantity model. Traditional crisp system cost observes that some costs may belong to the uncertain factors. It is necessary to extend the system cost to treat also the vague costs. We introduce a new concept which we call entropy and show that the total payoff satisfies the optimization property. We show how special case of this problem reduce to perfect results, and how post deteriorated discounted entropic order quantity model is a generalization of optimization. It has been imperative to demonstrate this model by analysis, which reveals important characteristics of discounted structure. Further numerical experiments are conducted to evaluate the relative performance between the fuzzy and crisp cases in EnOQ and EOQ separately.

Key Words: Discounted Selling Price, Fuzzy, Instant Deterioration, Inventory.

1. INTRODUCTION

In this paper we consider a continuous review, using fuzzy arithmetic approach to the system cost for perishable items. In traditional inventory models it has been common to apply fuzzy on demand rate, production rate and deterioration rate, whereas applying fuzzy arithmetic in system cost usually ignored in [5] and [13]. From practical experience, it has been found that uncertainty occurs not only due to lack of information but also as a result of ambiguity concerning the description of the semantic meaning of declaration of statements relating to an economic world. The fuzzy set theory was developed on the basis of non-random uncertainties. For this reason, we consider the same since no researcher have discussed EnOQ model by introducing the holding cost and disposal cost as the fuzzy number. The model provides an approach for quantifying these benefits which can be substantial, and should be reflected in fuzzy arithmetic system cost. Our objective is to find optimal values of the policy variables when the criterion is to optimize the expected total payoff over a finite horizon.

In addition, the product perishability is an important aspect of inventory control. Deterioration in general, may be considered as the result of various effects on stock, some of which are damage, decreasing usefulness and many more. While kept in store fruits, vegetables, food stuffs, bakery items etc. suffer from depletion by decent spoilage. Lot of articles are available in inventory literature considering deterioration. Interested readers may consult the survey paper of [10], [18], [16], [15], [4] and [9] classified perishability and deteriorating inventory models into two major categories, namely decay models and finite lifetime models. Products which deteriorate from the very beginning and the products which start to deteriorate after a certain time. Lot of articles are available in inventory literature considering deterioration. If this product starts to deteriorate as soon as it is received in the stock, then there is no option to provide pre-deterioration discount. Only we may give post deterioration discount on selling price.

Every organisation dealing with inventory faces a numbers of fundamental problems. Pricing decision is one of them. In the development of an EOQ system, we usually omit the case of discounting on selling price. But in real world, it exists and is quite flexible in nature. On the other hand, in order to motivate customers to order more quantities for instant deterioration model usually supplier offers discount on selling prices. [2] developed an inventory model under continuous discount pricing. [11] studied an inventory problem under the condition that multiple discounts can be used to sell excess inventory. [14] mentioned that discount is considered temporarily for exponentially decaying inventory model. However, most of the studies except few, do not attempt to unify the two research streams: temporary price reductions and instant deterioration. This paper outlines the issue in details.

The awareness of the importance of including entropy cost is increasing everyday. Indeed, entropy is frequently defined as the amount of disorder in a system. The above consideration leads us to some important points. First, the use of entropy must be carefully planned, taking into account the multiplicity of objectives inherent in this kind of decision problem. Second, these are several economic strategies with conflicting objectives in this kind of decision making process. [12] proposed an analogy between the behaviour of production system and the behaviour of physical system. The main purpose of this research is to introduce the concept of entropy cost to account for hidden cost such as the additional managerial cost that is needed to control the improvement process.

In last two decades the variability of inventory level dependent demand rate on the analysis of inventory system was described by researchers like [17], [1] and [3]. They described the demand rate as the power function of on hand inventory. There is a vast literature on stock development inventory and its outline can be found in the review article by [19] where he unified two types of inventory level dependent demand by considering a periodic review model. Researchers such as [1], [16], [17], [4], [6] and [8] discussed the EOQ model assuming time value of money, demand rate, deterioration rate, shortages and so on a constant or probabilistic number or an exponential function. In this paper we consider demand as a constant function for instant deterioration model.

The paper tackles to investigate the effect of the approximation made by using the average payoff when determining the optimal values of the policy variables. The problem consists of the simultaneous optimization of fuzzy entropic EOQ and crisp entropic EOQ model, taking into account the conflicting payoffs of the different decision makers involved in the process. A policy iteration algorithm is designed with the help of [7] and optimum solution is obtained through LINGO software. In order to make the comparisons equitable a particular evaluation function based on discount is suggested. Numerical experiments are carried out to analyse the magnitude of the approximation error. However, a discount during post deterioration time, fuzzy system cost which might lead to a non-negligible approximation errors. The remainder of this paper is organised as follows. In section 2 assumptions and notations are provided for the development of the model. Section 3 describes the model formulation. Section 4 develops the fuzzy model. Section 5 provides mathematical analysis. In section 6, an illustrative numerical experiment is given to illustrate the procedure of solving the model. Finally section 7 concludes this article with a brief summary and provides some suggestions for future research.

TABLE-1: Major Characteristics of Inventory Models on Selected Researches.

Author(s) and published Year	Structure of the Model	Deterioration	Inventory Model Based on	Discount allowed	Demand	Back-logging allowed
Mahata et al. (2006)	Fuzzy	Yes (constant)	EOQ	No	Constant	No
Panda et al. (2009)	Crisp	Yes (constant)	EOQ	Yes	Stock dependent	Yes (partial)
Jaber et al. (2008)	Crisp	Yes (on hand inventory)	EnOQ	No	Unit selling price	No
Vujosevic et al. (1996)	Fuzzy	No	EOQ	No	Constant	No
Skouri et al. (2007)	Crisp	Yes (Weibull)	EOQ	No	Ramp	Yes (partial)
Present paper (2010)	Fuzzy	Yes (constant)	EnOQ	Yes	Constant	No

2. NOTATIONS AND ASSUMPTIONS

Notations

C0 : set up cost
c : per unit purchase cost of the product
s : constant selling price of the product per unit (s>c)
h : holding cost per unit per unit time
d : disposal cost per unit.
r : discount offer per unit after deterioration.
Q1 : order level for post deterioration discount on selling price with instant deterioration.
Q2 : order level for no discount on selling price with instant deterioration.
T1, T2 : cycle lengths for the above two respective cases.

Assumptions

Replenishment rate is infinite.
The deterioration rate θ is constant and (0 <θ < 1)
3. Demand is constant and defined as follows.
R (I (t)) = a
Where a>0 is the demand rate independent of stock level.
r, $(0 \le r \le 1)$ is the percentage discount offer on unit selling price during instant deterioration.
$\alpha = (1-r)^{-n}$ ($n \in R$ the set of real numbers) is the effect of discounting selling price on demand during deterioration. α is determined from priori knowledge of the seller with constant demand.

5. The entropy generation must satisfy $S = \frac{d\sigma(t)}{dt}$ where, $\sigma(t)$ is the total entropy generated by time t and S is the rate at which entropy is generated. The entropy cost is computed by dividing the total commodity flow in a cycle of duration Ti. The total entropy generated over time Ti as $\sigma(T_i) = \int_O^{T_i} S dt,\ S = \frac{R(I(t))}{s} = \frac{a}{s}$

Entropy cost per cycle is

EC (Ti) = (EC) With deterioration $= \frac{Q_i}{\sigma(T_i)}$ (i=1,2)

3. MATHEMATICAL MODEL

At the beginning of the replenishment cycle the inventory level raises to Q1. As the time progresses it is decreased due to instantaneous stock with constant demand. Ultimately inventory reaches zero level at T1. As instant deterioration starts from origin, r% discount on selling price is provided to enhance the demand of decreased quality items. This discount is continued for the rest of the replenishment cycle. Then the behaviour of inventory level is governed by the following system of linear differential equation.

$$\frac{dI(t)}{dt} = -[\alpha a + \theta\, I(t)] \quad 0 \le t \le T_1 \tag{1}$$

with the initial boundary condition

$$\left.\begin{array}{l} I(0) = Q_1 \\ \text{and} \quad I(T_1) = 0 \end{array}\right\} 0 \le t \le T_1$$

Solving the equations,

$$I(t) = \frac{a\alpha}{\theta}\left[e^{\theta(T_1 - t)} - 1\right] \quad 0 \le t \le T_1 \tag{2}$$

$$Q_1 = \frac{a\alpha}{\theta}\left[e^{\theta T_1} - 1\right] \tag{3}$$

Holding cost and disposal cost of inventories in the cycle is,

$$HC + DC = (h + \theta d)\int_0^{T_1} I(t)\, dt$$

Purchase cost in the cycle is given by PC = cQ1.

Entropy cost in the cycle is

EC=(EC)With deterioration= $\dfrac{Q_{i\ with\,deterioration}}{\sigma(T_1)} = \dfrac{Q_1}{\sigma(T_1)}$

$$\sigma(T_1) = \int_0^{T_1} S dt = \int_0^{T_1} \frac{a}{s} dt = \frac{aT_1}{s}; \quad EC = \frac{sQ_1}{aT_1}$$

Total sales revenue in the order cycle can be found as

$$SR = s\left[\alpha(1-r)\int_0^{T_1} a\ dt\right]$$

Thus total profit per unit time of the system is

$$\pi_1(r,\ T_1) = \frac{TP_1}{T_1}$$

$$= \frac{1}{T_1}\left[SR - PC - HC - DC - EC - OC\right]$$

On integration and simplification of the relevant costs, the total profit per unit time becomes

$$\pi_1 \quad = \frac{1}{T_1}\left[s\alpha(1-r)aT_1 - h\left[\frac{e^{\theta T_1}-1}{\theta} - T_1\right]\frac{a\alpha}{\theta} - \frac{\theta d a\alpha}{\theta}\left[\frac{e^{\theta T_1}-1}{\theta} - T_1\right] - \frac{sQ_1}{aT_1} - cQ_1 - C_0\right] \tag{4}$$

If the product starts to deteriorate as soon as it is received in the stock, then there is only one option we may give post deterioration discount. The post deterioration discount on selling price is to be given in such a way that the discounted selling price is not less that the unit cost of the product, i.e. s(1-r)-c>0. Applying this constraint on unit total profit function we have the following maximization problem.

$$\begin{aligned} &\text{Maximize} && \pi_1(r, T_1) \\ &\text{Subject to} && r < 1 - \frac{c}{s} \\ & && \forall\, r, T_1 \geq 0 \end{aligned} \tag{5}$$

$$\pi_1 = F_1 + F_2 h + F_3 d$$

where $F_1 = \dfrac{1}{T_1}\left[s\alpha(1-r)aT_1 - \dfrac{sQ_1}{aT_1} - cQ_1 - C_0\right]$ (6)

$$F_2 = -\frac{1}{T_1}\left[\frac{e^{\theta T_1}-1}{\theta} - T_1\right]\frac{a\alpha}{\theta} \tag{7}$$

$$F_3 = -\frac{1}{T_1}\left[a\alpha\left(\frac{e^{\theta T_1}-1}{\theta} - T_1\right)\right] \tag{8}$$

4. FUZZY MODEL

We replace the holding cost and disposal cost by fuzzy numbers $\tilde{h}$ and $\tilde{d}$ respectively. By expressing $\tilde{h}$ and $\tilde{d}$ as the normal triangular fuzzy numbers (h1, h0, h2) and (d1, do, d2) , where, h1=h-Δ_1, ho = h, h2= h+Δ_2 , $d_1 = d - \Delta_3,\ d_0 = d,\ d_2 = d + \Delta_4$ such that

$0<\Delta_1<h,\ 0<\Delta_2, 0<\Delta_3<d,\ \ 0<\Delta_4, \Delta_1, \Delta_2, \Delta_3$ and Δ_4 are determined by the decision maker based on the uncertainty of the problem.

The membership function of fuzzy holding cost and fuzzy disposal cost are considered as:

$$\mu_{\tilde h}(h)=\begin{cases}\dfrac{h-h_1}{h_0-h_1},\ h_1\le h\le h_0\\ \dfrac{h_2-h}{h_2-h_0},\ h_0\le h\le h_2\\ 0 \qquad ,\ otherwise\end{cases} \tag{9}$$

$$\mu_{\tilde d}(d)=\begin{cases}\dfrac{d-d_1}{d_0-d_1},\ d_1\le d\le d_0\\ \dfrac{d_2-d}{d_2-d_0},\ d_0\le d\le d_2\\ 0 \qquad ,\ otherwise\end{cases} \tag{10}$$

Then the centroid for $\tilde h$ and $\tilde d$ are given by

$$M_{\tilde h}=\frac{h_1+h_o+h_2}{3}=h+\frac{\Delta_2-\Delta_1}{3} \text{ and } M_{\tilde d}=\frac{d_1+d_o+d_2}{3}=d+\frac{\Delta_4-\Delta_3}{3} \text{ respectively.}$$

For fixed values of r and T_1, let

$$Z(h,d)=F_1(r,T_1)+F_2(r,T_1)h+F_3(r,T_1)d=y$$

Let $h=\dfrac{y-F_1-F_3d}{F_2}$, $\dfrac{\Delta_2-\Delta_1}{3}=\psi_1$ and $\dfrac{\Delta_4-\Delta_3}{3}=\psi_2$

By extension principle the membership function of the fuzzy profit function is given by

$$\mu_{\tilde z(\tilde h,\tilde d)}(y)=\underset{(h,d)\in Z^{-1}(y)}{Sup}\left\{\mu_{\tilde h}(h)\vee\mu_{\tilde d}(d)\right\}$$

$$=\underset{d_1\le d\le d_2}{Sup}\left\{\mu_{\tilde h}\left(\frac{y-F_1-F_3d}{F_2}\right)\vee\mu_{\tilde d}(d)\right\} \tag{11}$$

Now,

$$\mu_{\tilde h}\left(\frac{y-F_1-F_3d}{F_2}\right)=\begin{cases}\dfrac{y-F_1-F_2h_1-F_3d}{F_2(h_0-h_1)} &,\ u_2\le d\le u_1\\ \dfrac{F_1+F_2h_2+F_3d-y}{F_2(h_2-h_0)} &,\ u_3\le d\le u_2\\ 0 &,\ otherwise\end{cases} \tag{12}$$

where,

$$u_1=\frac{y-F_1-F_2h_1}{F_3},\ u_2=\frac{y-F_1-F_2h_0}{F_3} \text{ and } u_3=\frac{y-F_1-F_2h_2}{F_3}$$

when $u_2\le d$ and $d\le u_1$ then $y\le F_1+F_2h_0+F_3d_0$ and $y\ge F_1+F_2h_1+F_3d_1$. It is clear that for every $y\in[F_1+F_2h_1+F_3d_1\ ,\ F_1+F_2h_0+F_3d_0],\mu_y(y)=PP'$. From the equations (9) and (12) the value of PP' may be found by solving the following equation:

$$\frac{d-d_1}{d_0-d_1}=\frac{y-F_1-F_2h_1-F_3d}{F_2(h_0-h_1)}$$

$$d=\frac{(y-F_1-F_2h_1)(d_0-d_1)+F_2d_1(h_0-h_1)}{F_2(h_0-h_1)+F_3(d_0-d_1)}$$

or

Therefore, $$PP'=\frac{d-d_1}{d_0-d_1}=\frac{y-F_1-F_2h_1-F_3d}{F_2(h_0-h_1)+F_3(d_0-d_1)}=\mu_1(y)$$, (say). (13)

When $u_3 \le d$ and $d \le u_2$ then $y \le F_1+F_2h_2+F_3d_2$ and $y \ge F_1+F_2h_0+F_3d_0$. It is evident that for every $y\in[F_1+F_2h_0+F_3d_0\ ,\ F_1+F_2h_2+F_3d_2],\mu_{\tilde{y}}(y)=PP''$. From the equations (9) and (12), the value of PP'' may be found by solving the following equation:

$$\frac{d_2-d}{d_2-d_0}=\frac{F_1+F_2h_2+F_3d-y}{F_2(h_2-h_0)}$$

$$d=\frac{F_2d_2(h_2-h_0)-(F_1+F_2h_2-y)(d_2-d_0)}{F_2(h_2-h_0)+F_3(d_2-d_0)}$$

or,

Therefore, $$PP''=\frac{d_2-d}{d_2-d_0}=\frac{F_1+F_2h_2+F_3d_2-y}{F_2(h_2-h_0)+F_3(d_2-d_0)}=\mu_2(y)$$, (say). (14)

Thus the membership function for fuzzy total profit is given by

$$\mu_{\tilde{z}(\tilde{h},\tilde{d})}(y)=\begin{cases}\mu_1(y); & F_1+F_2h_1+F_3d_1\le y\le F_1+F_2h_0+F_3d_0\\ \mu_2(y); & F_1+F_2h_0+F_3d_0\le y\le F_1+F_2h_2+F_3d_2\\ 0\ ; & \text{otherwise}\end{cases} \quad (15)$$

Now, let $P_1=\int_{-\infty}^{\infty}\mu_{\tilde{z}(\tilde{h},\tilde{d})}(y)$ dy and $R_1=\int_{-\infty}^{\infty}y\,\mu_{\tilde{z}(\tilde{h},\tilde{d})}(y)$ dy

Hence, the centroid for fuzzy total profit is given by

$$\begin{aligned}\tilde{\pi}_1 &= M_{\tilde{TP}}(r,T_1)=\frac{R_1}{P_1}\\ &= F_1(r,T_1)+F_2(r,T_1)h+F_3(r,T_1)d\\ &\quad+\psi_1F_2(r,T_1)+\psi_2F_2(r,T_1)\end{aligned} \quad (16)$$

$$M_{\tilde{TP}}(r,T_1)=F_1+(h+\psi_1)F_2+(d+\psi_2)F_3 \quad (17)$$

where, $F_1(r,T_1)$, $F_2(r,T_1)$ and $F_3(r,T_1)$ are given by equations (6), (7) and (8).

The post-deterioration discount on selling price is to be given in such a way that the discounted selling price is not less that the unit cost of the product, i.e. s(1-r)-c>0.

Applying this constraint on the unit total profit function in equation (17) we have the following maximization problem.

Maximize $M_{\tilde{TP}_1}(r,T_1)$

Subject to, $r<1-\frac{c}{s}$ (18)

$\forall \quad r,T_1\ge 0$

Our objective here is to determine the optimal values of r and T_1 to maximize the unit profit function. It is very difficult to derive the results analytically. Thus some numerical methods must be applied to derive the optimal values of r and T_1, hence the unit profit function. There are several methods to cope with constraint optimization problem numerically. But here we use penalty function method [7] and LINGO software to derive the optimal values of the decision variables.

a. Special Case

b. I Model for instant deterioration with no discount

In this case order level and unit profit function for model with constant deterioration and constant demand with no discount are obtained from (3) and (4) by substituting r=0 as

$$Q_2 = \frac{a}{\theta}\left(e^{\theta T_2} - 1\right) \quad (19)$$

From equation (4) total profit per unit time becomes

$$\pi_2(T_2) = \frac{TP_2}{T_2} = \frac{1}{T_2}\left[saT_2 - (h+\theta d)\frac{a}{\theta}\left[\frac{e^{\theta T_2}-1}{\theta} - T_2\right] - \frac{sQ_2}{aT_2} - cQ_2 - C_0\right] \quad (20)$$

$$= F_4 + F_5 h + F_6 d$$

where,

$$F_4 = \frac{1}{T_2}\left[saT_2 - \frac{sQ_2}{aT_2} - cQ_2 - C_0\right] \quad (21)$$

$$F_5 = \frac{-a}{\theta T_2}\left[\frac{e^{\theta T_2}-1}{\theta} - T_2\right] \quad (22)$$

$$F_6 = -\frac{a}{\theta T_2}\left[\frac{e^{\theta T_2}-1}{\theta} - T_2\right] \quad (23)$$

Thus we have to determine T_2 from the fuzzy maximization problem

$$\text{maximize} \quad M_{\tilde{TC}_2}(T_2) \quad (24)$$

$$\forall \quad T_2 \geq 0$$

where, $M_{\tilde{TC}_2}(T_2) = F_4 + (h+\psi_1)F_5 + (d+\psi_2)F_6 = \tilde{\pi}_2$. (25)

5. MODEL ANALYSIS THEOREM

$$\text{For } n \neq 1, \tilde{\pi}_1 > \tilde{\pi}_2 \text{ if } r < \min\left\{1-\frac{c}{s}, 1-\frac{n\left(c+\frac{s}{aT_2}\right)}{s(n-1)}\right\}.$$

Proof:

The values of $\tilde{\pi}_1$ for fixed r are always less than optimal value of r. Thus it is sufficient to show that $\tilde{\pi}_1 > \tilde{\pi}_2$ for fixed r. Here, T_1 is the cycle length when post deterioration discount is applied on unit selling price to enhance the demand of decreased quality items. For the enhancement of demand the inventory depletion rate will be higher and consequently the cycle time will reduce. T2 is the cycle length when no discount is applied on selling price. Obviously T2 is greater than T1. Without loss of generality let both the profit function $\tilde{\pi}_1$ and $\tilde{\pi}_2$ are positive.

$$\tilde{\pi}_1 - \tilde{\pi}_2 = \frac{\tilde{TP}_1}{T_1} - \frac{\tilde{TP}_2}{T_2} \geq \frac{\tilde{TP}_1 - \tilde{TP}_2}{T_2}$$

It is sufficient to show that $\frac{\tilde{TP_1}-\tilde{TP_2}}{T_2}>0$. If it can be shown that $\frac{\tilde{TP_1}-\tilde{TP_2}}{T_2}>0$ is an increasing function of r then our purpose will be served. Now differentiating it with respect to r we have,

$$\frac{\partial(\tilde{\pi}_1-\tilde{\pi}_2)}{\partial r}=\frac{1}{T_2}\left[\left(s(n-1)(1-r)+n\frac{h+\theta d}{\theta}\right)\frac{aT_2}{(1-r)^{n+1}}+\left[-c-\frac{h+\theta d}{\theta}-\frac{s}{aT_2}\right]\frac{an\left(e^{\theta T_2}-1\right)}{\theta(1-r)^{n+1}}\right]$$

Therefore, $\frac{\tilde{TP_1}-\tilde{TP_2}}{T_2}>0 \Rightarrow \frac{\partial(\tilde{\pi}_1-\tilde{\pi}_2)}{\partial r}>0$,

i.e. if

$$\left[s(n-1)(1-r)+n\frac{h+\theta d}{\theta}\right]+\left[-c-\frac{h+\theta d}{\theta}-\frac{s}{aT_2}\right]\times\frac{n\left(e^{\theta T_2}-1\right)}{\theta T_2}>0 \tag{26}$$

Now, $\frac{\left(e^{\theta T_2}-1\right)}{\theta T_2}>1.$

we have,

$$s(n-1)(1-r)+n\left[-c-\frac{s}{aT_2}\right]>0$$

i.e. $$r<\frac{s(n-1)+n\left[-c-\frac{s}{aT_2}\right]}{s(n-1)}$$

We have the restriction $r<1-\frac{c}{s}$.

Therefore, $\tilde{\pi}_1>\tilde{\pi}_2$ if

$$r<\min\left\{1-\frac{c}{s},1-\frac{n\left(c+\frac{s}{aT_2}\right)}{s(n-1)}\right\} \tag{27}$$

Theorem indicates that for n≠1 post instant deterioration discount on unit selling price produces higher profit than that of instant deterioration with no discount on unit selling price in fuzzy environment, if the percentage of post deterioration discount on unit selling price is less than min $\left\{1-\frac{c}{s},1-\frac{n\left(c+\frac{s}{aT_2}\right)}{s(n-1)}\right\}$.

A simple managerial indication is that in pure inventory scenario if the product deteriorates after a certain time then it is always more profitable to apply only post deterioration discount on unit selling price and the amount of percentage discount must be less than the limit provided in equation (27) for the post deterioration discount.

6. NUMERICAL EXAMPLE

LINGO software is used to solve the aforesaid numerical example.

We redo the same example of [18] to see the optimal replenishment policy while considering the fuzzy holding cost, fuzzy disposal cost and entropy cost. The parameter values are a=80, b=0.3, h=0.6, d=2.0, s=10.0, C0=100.0, c=4.0, θ=0.03, n =2.0, Δ1=0.1, Δ2=0.2, Δ3=0.5, Δ4=0.8.

After 185 and 50 iterations in Table 2 we obtain the optimal replenishment policy for instant deterioration fuzzy entropic order quantity models with post deterioration discount and no discount respectively. The total profits for both the cases obtained here is at least 4.12% and 3.76% respectively less than that in [18] i.e. our CEOQ models. This is because we modified the model by introducing the hidden cost that is entropy cost where the optimal values for both the cases are 21.03623 and 20.28649 respectively. In Tables 3 and 4 we obtain the numerical results of different models like FEnOQ, FEOQ, CEnOQ and CEOQ for above two cases separately. The behaviour of the total profit to the lot size and the cycle length of post deterioration discounted model is shown in Figure 1.

TABLE-2: The Numerical Results of the Instant Deterioration Fuzzy Entropic Order Quantity (FEnOQ) Models (i=1,2)

Model	Local optimal solution found at iteration	r	Ti	Qi	EC	π_i
FEnOQ (Only post deterioration discount)	185	0.0350	1.8221	160.8798	21.0362	354.1393
FEnOQ (No discount)	50	-	1.8814	154.8204	20.28649	353.6979
% change	-	-	-3.1457	3.9136	3.6958	0.1248

TABLE-3: Comparison of Results for the different Post Deterioration Discount Models

Model	Local optimal solution found at iteration	r	Ti	Qi	EC	π_1
FEnOQ	185	0.0350	1.8221	160.8798	21.0362	354.1393
FEOQ	193	0.0673	1.6063	151.3270	-	366.6226
CEnOQ	105	0.0392	1.8561	165.4009	21.1392	357.0641
CEOQ	196	0.0708	1.6367	155.4112	-	369.3739

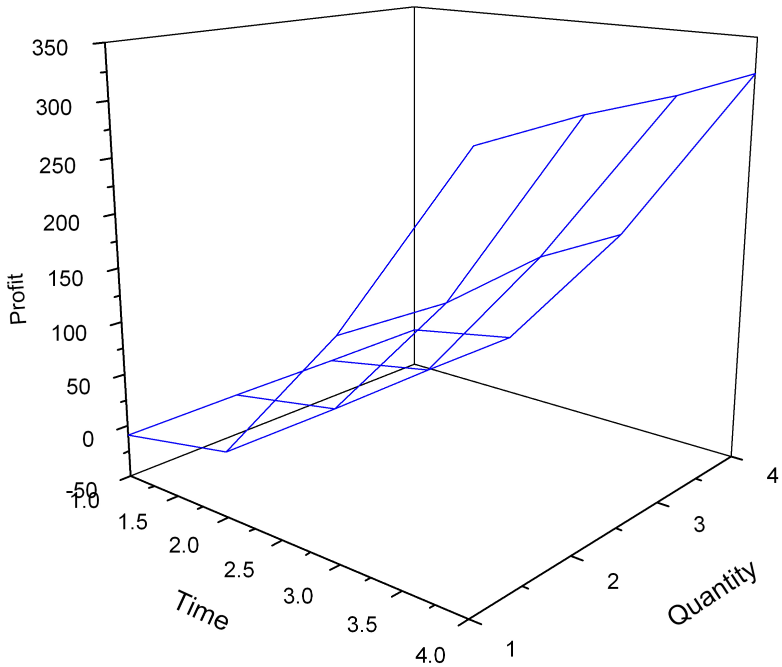

FIGURE1: The behaviour of the total profit to the lot size and the cycle length of post deterioration discounted model.

7. COMPARATIVE EVALUATION

Table 2 shows that 3.4% discount on post deterioration model is provided on unit selling price to earn 0.12% more profit than that with no discounted instant deterioration model. From Table 5 it indicates that the uncertainty and entropy cost are provided on the post deterioration discount model to lose 3.4%, 0.81% and 4.12% less profits for FEOQ, CEnOQ and CEOQ models respectively than that with FEnOQ model. Similarly it shows that the no discounted deterioration model to lose 3.08%, 0.78% and 3.76% less profits for FEOQ, CEnOQ and CEOQ models respectively than that with FEnOQ model. This paper investigates a computing schema for the EOQ in fuzzy sense. From Tables 3 and 4 it shows that the fuzzy and crisp results are very approximate, i.e. it permits better use of EOQ as compared to crisp space arising with the little change in holding cost and in disposal cost respectively. It indicates the consistency of the crisp case from the fuzzy sense.

TABLE-4: Comparison of Results for the different No Discounted Instant Deterioration Models

Model	Local optimal solution found at iteration	T2	Q2	EC	π_2
FEnOQ	50	1.8814	154.8207	20.28649	353.6979
FEOQ	32	1.7203	141.2193	-	364.9558
CEnOQ	48	1.9239	158.4196	20.2931	356.5054
CEOQ	34	1.7592	144.4996	-	367.5178

TABLE-5: Relative Error (RE) of Post Deterioration Discount and No Discounted Deterioration FEnOQ Models with the different Models

FEnOQ	Q1	160.8798	Q2	154.8207
	π_1	354.1393	π_2	353.6979
FEOQ	Q11	151.3270	Q21	141.2193
	π_{11}	366.6226	π_{21}	364.9558
RE	% change	6.3127	% change	9.6314
	% change	-3.4050	% change	-3.0847
CEnOQ	Q12	165.4009	Q22	158.4196
	π_{12}	357.0641	π_{22}	356.5054
RE	% change	-2.7334	% change	-2.2718
	% change	-0.8191	% change	-0.7878
CEOQ	Q13	155.4112	Q23	144.4996
	π_{13}	369.3739	π_{23}	367.5178
RE	% change	3.5188	% change	-6.6665
	% change	-4.1244	% change	-3.7603

8. CRITICAL DISCUSSION

When human originated data like holding cost and disposal cost which are not precisely known but subjectively estimated or linguistically expressed is examined in this paper. The mathematical model is developed allowing post deterioration discount on unit selling price in fuzzy environment. It is found that, if the amount of discount is restricted below the limit provided in the model analysis, then the unit profit is higher. It is derived analytically that the post deterioration discount on unit selling price is to earn more revenue than the revenue earned for no discount model. The numerical example is presented to justify the claim of model analysis. Temporary price discount for perishable products to enhance inventory depletion rate for profit maximization is an area of interesting research. This paper introduces the concept of entropy cost to account for hidden cost such as the additional managerial cost that is needed to control the improvement of the process. This paper examines the idea by extending the analysis of [18] by introducing fuzzy approach and entropy cost to provide a firm its optimum discount rate, replenishment schedule, replenishment order quantity simultaneously in order to achieve its maximum profit.

Though lower amount of percentage discount on unit selling price in the form of post deterioration discount for larger time results in lower per unit sales revenue, still it is more profitable. Because the inventory depletion rate is much higher than for discount with enhanced demand resulting in lower amount inventory holding cost and deteriorated items. Thus it can be conjectured that it is always profitable to apply post deterioration discount on unit selling price to earn more profit. Thus the firm in this case can order more to get earn more profit.

These models can be considered in a situation in which the discount can be adjusted and number of price changes can be controlled. Extension of the proposed model to unequal time price changes and other applications will be a focus of our future work.

9. CONCLUSION

This paper provides an approach to extend the conventional system cost including fuzzy arithmetic approach for perishable items with instant deterioration for the discounted entropic order quantity model in the adequacy domain. To compute the optimal values of the policy parameters a simple and quite efficient policy model was designed. Theorem determines effectively the optimal discount rate r for post deterioration discount. Finally, in numerical experiments the solution from the instant deteriorated model evaluated and compared to the solutions of other different EnOQ and traditional EOQ policies.

However, we saw few performance differences among a set of different inventory policies in the existing literature. Although there are minor variations that do not appear significant in practical terms, at least when solving the single level, incapacitated version of the lot sizing problem. From our analysis it is demonstrated that the retailer's profit is highly influenced by offering post discount on selling price. The results of this study give managerial insights to decision maker developing an optimal replenishment decision for instant deteriorating product. Compensation mechanism should also be included to induce collaboration between retailer and dealer in a meaningful supply chain. We conclude this paper by summarizing some of the managerial insights resulting from our work.

In general, for normal parameter values the relative payoff differences seem to be fairly small. The optimal solution of the suggested post deterioration discounted model has a higher total payoff as compared with no discounted model. Conventional wisdom suggests that workflow collaboration in a fuzzy entropic model in a varying deteriorating product in market place are promising mechanism and achieving a cost effective replenishment policy. Theoretically such extensions would require analytical paradigms that are considerably different from the one discussed in this paper, as well as additional assumptions to maintain tractability.

The approach proposed in the paper based on EnOQ model seems to be a pragmatic way to approximate the optimum payoff of the unknown group of parameters in inventory management problems. The assumptions underlying the approach are not strong and the information obtained seems worthwhile. Investigating optimal policies when demand are generated by other process and designing models that allow for several orders outstanding at a time, would also be challenging tasks for further developments. Its use may restrict the model's applicability in the real world. Future direction may be aimed at considering more general deterioration rate or demand rate. Uses of other demand side revenue boosting variables such as promotional efforts are potential areas of future research. There are numerous ways in which one could consider extending our model to encompass a wider variety of operating environments. The proposed paper reveals itself as a pragmatic alternative to other approaches based on constant demand function with very sound theoretical underpinnings but with few possibilities of actually being put into practice. The results indicate that this can become a good model and can be replicated by researchers in neighbourhood of its possible extensions. As regards future research, one other line of development would be to allow shortage and partial backlogging in the discounted model.

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A Customizable Model of Head-Related Transfer Functions Based on Pinna Measurements

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Abstract

This paper proposes a method to model Head-Related Transfer Functions (HRTFs) based on the shape and size of the outer ear. Using signal processing tools, such as Prony's signal modeling method, a dynamic model of the pinna has been obtained, that completes the structural model of HRTFs used for digital audio spatialization.

Listening tests conducted on 10 subjects showed that HRTFs created using this pinna model were 5% more effective than generic HRTFs in the frontal plane. This model has been able to reduce the computational and storage demands of audio spatialization, while preserving a sufficient number of perceptually relevant spectral cues.

Keywords: HRTF, Binaural, HRIR, Pinna, Model

1. INTRODUCTION

HRTFs represent the transformation undergone by the sound signals, as they travel from their source to both of the listener's eardrums. This transformation is due to the interaction of sound waves with the torso, shoulder, head and outer ear of a listener [9]. Therefore, the two components of these HRTF pairs (left and right) are typically different from each other, and pairs corresponding to sound sources at different locations around the listener are different. Furthermore, since the physical elements that determine the transformation of the sounds reaching the listener's eardrums (i.e., the listener's head, torso and pinnae), are somewhat different for different listeners, and so should be their HRTF sets [2].

Currently, some spatialization systems make use of HRTFs that are empirically measured for each prospective user. These "custom" HRTFs are anthropometrically correct for each user, but the equipment, facilities and expertise required to obtain these "measured HRTF pairs", constrain their application to high-end, purpose-specific sound spatialization systems only [2]. For most consumer-grade applications, sound spatialization systems resort to the use of "generic" transfer functions, measured from a manikin with "average" physical characteristics [7], which, evidently is a fundamentally imperfect approach.

This paper reports on our work to advance an alternative approach to sound spatialization, based on the postulation of anthropometrically-related "structural models" [6] that will transform a single-channel audio signal into a left/right binaural spatialized pair, according to the sound source simulation. Specifically, the work reported here proposes linkages between the parameters of the HRTF model and key anthropometric features of the intended listener's pinna, so that the model, and consequently the resulting HRTFs are easily "customizable" according to a small set of anthropometric measurements.

2. MEASUREMENTS AND IMPLEMENTATION

Current sound spatialization systems use HRTFs, represented by their corresponding impulse response sequences, the Head-Related Impulse Responses, (HRIRs) to process, by convolution, a single-channel digital audio signal, resulting in the two components (left and right) of a binaural spatialized sound. When these two channels are delivered to the listener through headphones, the sound will seem to emanate from the source location corresponding to the HRIR pair used for the spatialization process [4].

In our laboratory, we use the Ausim3D's HeadZap HRTF Measurement System [1]. This system measures a 256-point impulse response for both the left and the right ear using a sampling frequency of 96 KHz. Golay codes are used to generate a broad-spectrum stimulus signal delivered through a Bose Acoustimass speaker. The response is measured using miniature blocked meatus microphones placed at the entrance to the ear canal on each side of the head. Under control of the system, the excitation sound is issued and both responses (left and right ear) are captured. Since the Golay code sequences played are meant to represent a broad-band excitation equivalent to an impulse, the sequences captured in each ear are the impulse responses corresponding to the HRTFs. The system provides these measured HRIRs as a pair of 256-point minimum-phase vectors, and an additional delay value that represents the Interaural Time Difference (ITD), i.e., the additional delay observed before the onset of the response collected from the ear that is farthest from the speaker position. In addition to the longer onset delay of the response from the "far" or "contralateral ear" (with respect to the sound source), this response will typically be smaller in amplitude than the response collected in the "near" or "ipsilateral ear". The difference in amplitude between HRIRs in a pair is referred to as the Interaural Intensity Difference (IID).

Our protocol records HRIR pairs from source locations at the 72 possible combinations of ϕ = {-36°, -18°, 0°, 18°, 36°, 54°} and θ = {0°, 30°, 60°, 90°, 120°, 150°, 180°, -150°, -120°, -90°, -60°, -30°}. The left (L) and right (R) HRIRs collected for a source location at azimuth □ and elevation ϕ are symbolized by $h_{L,\theta,\phi}$ and $h_{R,\theta,\phi}$, respectively. The corresponding HRTFs are $H_{L,\theta,\phi}$ and $H_{R,\theta,\phi}$. The creation of a spatialized binaural sound (left and right channels) involves convolving the single-channel digital sound to be spatialized, s(n), with the HRIR pair corresponding to the azimuth and elevation of the intended virtual source location:

$$y_{L,\theta,\phi}(n) = \sum_{k=-\infty}^{\infty} h_{L,\theta,\phi}(k) \cdot s(n-k) \quad , \text{ and } \quad y_{R,\theta,\phi}(n) = \sum_{k=-\infty}^{\infty} h_{R,\theta,\phi}(k) \cdot s(n-k) \qquad (1)$$

3. STRUCTURAL MODEL

Structural models of HRTF are based on the premise that each anthropometric feature of the listener affects the HRTF in a way that can be described mathematically [11]. Because such a model has its origin in the physical characteristics of the entities involved in the phenomenon, it should be possible to derive the value of its parameters (for a given source location), from the sizes of those entities, i.e., the anthropometric features of the intended listener. Proper identification of such parameters and adequate association of their numerical values with the anthropometric features of the intended listener may provide a mechanism to interactively adjust a generic base model to the specific characteristics of an individual. One of the most practical models has been proposed by Brown and Duda [6]. Their model is illustrated in Figure 1:

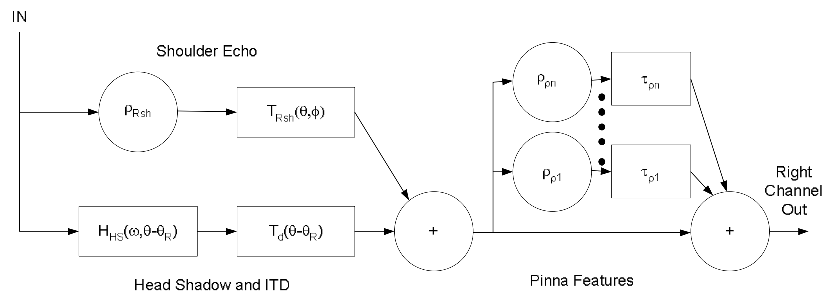

FIGURE1: Right Channel half for Brown & Duda's Structural HRTF Model. The model comprises a symmetric left half (not shown).

4. CUSTOMIZABLE PINNA MODEL

The Head-Shadow sub-model of Duda's structural model was developed by Lord Rayleigh and can be emulated by a one-pole/ one-zero model [5]. This model, incorporating the IID and ITD effects, can account for an approximate localization, particularly in the horizontal plane (elevation, $\phi = 0^o$). However, to produce elevation effects, a good pinna sub-model is required. The definition and anthropometric characterization of the pinna model has remained an open question, so far, and it is the objective of our work. Carlile [7] divides pinna models according to the main phenomenon that they address: Resonating, diffractive and reflective. From these, reflective models have attracted the most attention in the literature.

The intent of our work is to define a functional pinna sub-model that has anthropometric plausibility and then associate its parameters to anthropometric features of the listener's pinna. Taking into account the information available about the existence of a resonant effect implemented by the ear's concha [13] and according to the reflective pinna models discussed previously, we propose that the pinna may, in turn, be modeled as the series connection of an equivalent second-order resonator and a series of characteristic echoes, representing the delayed and attenuated secondary paths taken by the incoming sound, in addition to a "direct path". A block diagram representation of this model is shown in Figure 2.

It should be noted that this pinna model allows for the "direct-path", F0, and each one of the "echoes", F1, F2, and F3, to be affected by a different equivalent resonance.

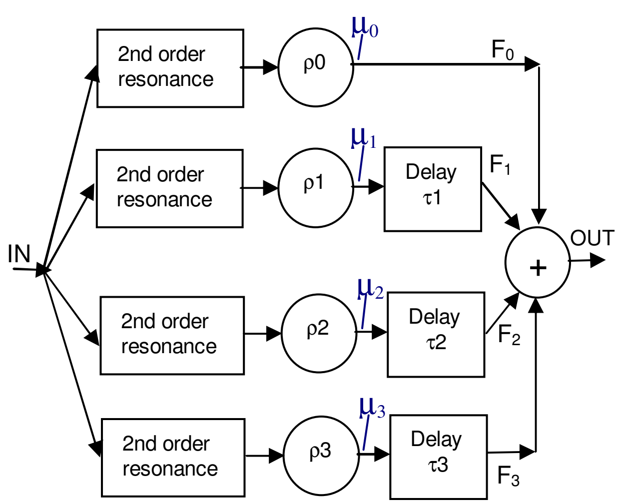

FIGURE 2: Proposed Pinna Model

Accordingly, HRIRs are envisioned as the impulse response of this model, which will be the superposition of four damped sinusoidals (the impulse responses of each of the 2nd order resonators), characterized by their frequency, f, and damping factor, σ. These damped sinusoidals are altered in their amplitude according to the ρ_k parameters, and delayed according to the τk parameters.

Thus, the instantiation of this proposed model will require the identification of the fk, σ_κ, ρ_κ and τ_κ values, to characterize the parameters of the model that successfully approximates an HRIR collected for a given azimuth and elevation, through the output provided by the pinna model. The main challenge in this operation is the fact that the several replicas of the damped oscillation are irreversibly mixed together, partially overlapping in time, in the measured HRIR. This problem was addressed by the sequential application of Prony's modeling algorithm [10, 12] to partial segments of the response. Prony's method approximates a given signal μ(t) as the superposition of p damped sinusoidals:

$$\mu(t) = \sum_{j=1}^{p} \rho_j e^{(\sigma_j t)} \sin(2\pi f_j t + \xi_j) \qquad (2)$$

Figure 3 shows the four F components, obtained using this method, from a measured HRIR

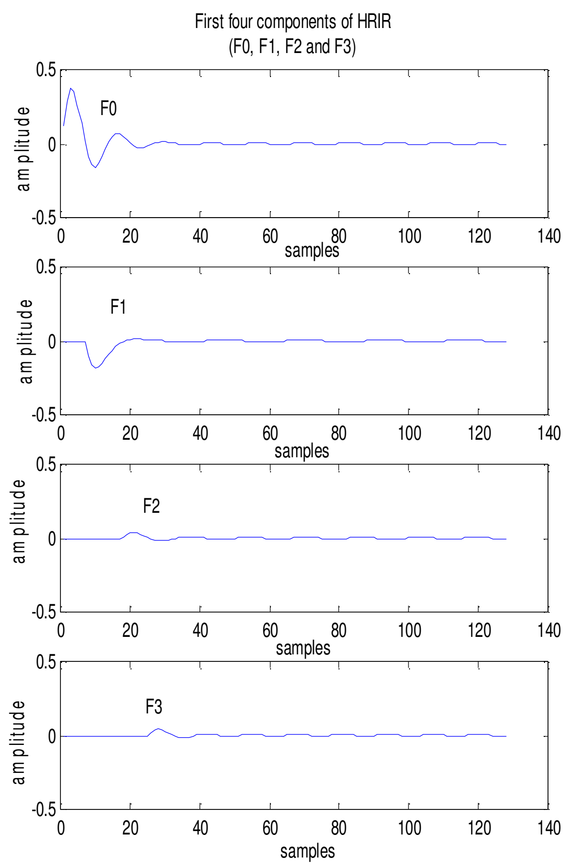

FIGURE 3: Example of HRIR Deconstruction

5. MEASUREMENT OF ANTHROPOMETRIC FEATURES

Key anthropometric features of the ears of the 15 experimental subjects in the study (same 15 subjects for whom the HRIRs were empirically measured with the Ausim 3D system) were captured by means of digital photography (including a distance reference), and laser 3-D scanning, using a Polhemus FastScan handheld scanner.

The features measured are: Ear length (EL), Ear width (EW), Concha width (CW), Concha height (CH), Helix length (HL), Concha area (CA), Concha volume (CV) and Concha depth (CD).

6. ASSOCIATION BETWEEN MODEL PARAMETERS AND ANTHROPOMETRIC MEASUREMENTS

Following the procedure described in the two preceding sections two independent sets of data were available for each pinna of each on of the 15 subjects in the study:

Estimated Model Parameters (for frontal plane sites):

$r_{0\phi}$, $\alpha_{0\,\phi}$, $\rho_{1\phi}$, $\rho_{2\,\phi}$, $\tau_{2\,\phi}$, $\rho_{3\,\phi}$, $\tau_{3\,\phi}$, $\rho_{4\,\phi}$ and $\tau_{4\,\phi}$, for $\phi = -36^o, -18^o, 0^o, 18^o, 36^o$, and 54^o

(Note, here r_0 and α_0 are the magnitude and angle of the poles of the resonator, which define the resonator response F0(n), in terms of its frequency f_0 and its damping factor σ_0)

Measured Anthropometric Features:
EL, EW, CH, CW, CA, CV, CD and HL

Under the assumption that the model parameters depend of the anthropometric features, a general dependency equation may be set, for each model parameter. For example, for the amplitude of the first replica in the pinna model, ρ_1, at $\phi = 54^o$, the following equation may be set up:

$$\rho_{0\phi=54} = KEL(EL) + KEW(EW) + KCH(CH) + KCW(CW) + KCA(CA) + KCV(CV) + KCD(CD) + KHL(HL) + B \quad (3)$$

Coalescing the data from both ears, at the same elevation, (under the assumption of symmetry), 30 equations like the one above can be set up, for each model parameter, at each elevation. Each group of 30 equations can then be analyzed through multiple regression to estimate the values of the constants (KEL, KEW,…KHL, B). The multiple regression analysis was carried out using the Statistical Package for the Social Sciences (SPSS).

7. MODEL EVALUATION AND RESULTS

Using the predictive equations found above, for each subject tested, a Model HRTF was created. Ultimately, the efficiency of the modeled sequences obtained by predicting the model parameters from the anthropometric measurements of the subjects was gauged in listening tests. In these tests, white noise bursts were spatialized using the modeled HRIR sequences, that had been obtained based on the ear measurements of the subject under test, for the six elevations under study. The order in which these elevations where used for the spatialization was randomized. Each elevation was simulated four times (i.e., there were 24 trials for each side of the head.) In each trial the subject would listen to each spatialized sound and then use a graphic user interface to indicate the perceived elevation. Since the spatialization was performed to emulate six specific locations, the absolute value of the angular difference between the perceived elevation and the emulated one would be considered as the elevation error for the trial. The subjects listened to the original, modeled and generic HRTFs. Figure 4 illustrates the average angular error (across all 10 subjects) experienced in the perception of the different emulated elevations for Original, Generic (B&K) and Model HRIRs. The global average error (across all subjects and all elevations) with the original HRTFs, was 23.7^o. The corresponding global average error with modeled HRIRs was 29.9^o. Finally, the global average error when the subjects used the generic HRIRs, collected from the B&K manikin, was 31.4^o.

It should be noted, however, that near the horizontal plane (e.g., between ϕ = -18^o and ϕ = 36^o), the performance of the modeled HRIRs was close to or better than, that of the individually measured HRIRs.

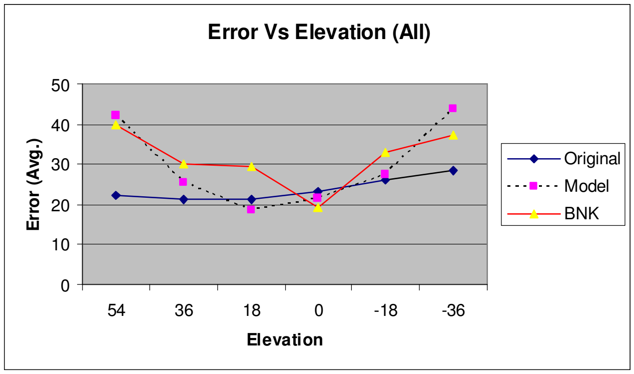

FIGURE 4: Localization performance using 3 different types of HRIRs.

8. CONCLUSIONS

This paper has presented a proposed functional model of the pinna, to be used as the output block in a structural HRTF model. Although this study resorted to the use of a relatively expensive 3-D laser scanner and specialized software to determine some of the anthropometric features of our subjects, which is a prerequisite to the use of the predictive equations developed in this research, it is likely that empirical relationships can be found to obtain these feature values from two-dimensional high-resolution photographs (commonly available) and a few direct physical measurements in the subject.

9. ACKNOWLEDGEMENT

This research was supported by NSF grants, HRD-0317692, IIS-0308155, and CNS-0426125.

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A Retail Category Inventory Management Model Integrating Entropic Order Quantity and Trade Credit Financing

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Abstract

A retail category inventory management model that considers the interplay of entropic product assortment and trade credit financing is presented. The proposed model takes into consideration of key factors like discounted-cash-flow. We establish a stylized model to determine the optimal strategy for an integrated supplier-retailer inventory system under the condition of trade credit financing and system entropy. This paper applies the concept of entropy cost estimated using the principles of thermodynamics.

The classical thermodynamics reasoning is applied to modelling such systems. The present paper postulates that the behaviour of market systems very much resembles those of physical systems. Such an analogy suggests that improvements to market systems might be achievable by applying the first and second laws of thermodynamics to reduce system entropy(disorder).

This paper synergises the above process of entropic order quantity and trade credit financing in an increasing competitive market where disorder and trade credit have become the prevailing characteristics of modern market system. Mathematical models are developed and numerical examples illustrating the solution procedure are provided.

Key words: Discounted Cash-Flow, Trade Credit, Entropy Cost.

1. INTRODUCTION

In the classical inventory economic order quantity (EOQ) model, it was tacitly assumed that the customer must pay for the items as soon as the items are received. However, in practice or when the economy turns sour, the supplier allows credit for some fixed time period in settling the payment for the product and does not charge any interest from the customer on the amount owned during this period.Goyal(1985)developed an EOQ model under the conditions of permissible delay in payments.Aggarwal and Jaggi(1995) extended Goyal's(1985) model to consider the deteriorating items .Chung(1999) presented an EOQ model by considering trade credit with DCF approach.Chang(2004) considered the inventory model having deterioration under inflation when supplier credits linked to order quantity. Jaber et al.(2008) established an entropic order quantity (EnOQ) model for deteriorating items by applying the laws of thermodynamics.Chung and Liao(2009) investigated an EOQ model by using a discounted-cash-flows(DCF) approach and trade credit depending on the quantity ordered.

The specific purpose of this paper is to trace the development of entropy related thought from its thermodynamic origins through its organizational and economic application to its relationship to discounted-cash-flow approach.

2. Model Development

2.1 Basis of the Model:

The classical economic order quantity (EOQ) or lot sizing model chooses a batch size that minimises the total cost calculated as the sum of two conflicting cost functions, the order/setup cost and the inventory holding costs. The entropic order quantity (EnOQ) is derived by determining a batch size that minimizes the sum of the above two cost and entropic cost.

Notations

T	:	the inventory cycle time, which is a decision variable;
C,A	:	the purchase cost and ordering cost respectively
h	:	the unit holding cost per year excluding interest change;
D	:	the demand rate per unit time.
D(T)	:	the demand rate per unit time where cycle length is T.
r	:	Discount rate (opportunity cost) per time unit.
Q	:	Procurement quantity;
M	:	the credit period;
W	:	quantity at which the delay payment is permitted;
σ(t)	:	total entropy generated by time t.
S	:	rate of change of entropy generated at time t.
E(t)	:	Entropy cost per cycle;
PV1(T)	:	Present value of cash-out-flows for the basic EnOQ model;
PV2(T)	:	present value of cash-out-flows for credit only on units in stock when $T \leq M$.
PV3(T)	:	Present value of cash-out-flows for credit only on units in stock when $T \geq M$.
$P(t), P_0)t)$	:	unit price and market equilibrium price at time t respectively.
$\mathrm{PV}_\infty(\mathrm{T})$	:	the present value of all future cash-flows.
T^*	:	the optimal cycle time of $\mathrm{PV}_\infty(\mathrm{T})$ when T>0.

ASSUMPTIONS

(1) The demand is constant.
(2) The ordering lead time is zero.
(3) Shortages are not allowed.
(4) Time period is infinite.
(5) If Q<W, the delay in payment is not permitted, otherwise, certain fixed trade credit period M is permitted. That is, Q<W holds if and only if T<W/D.
(6) During the credit period, the firm makes payment to the supplier immediately after use of the materials. On the last day of the credit period, the firms pays the remaining balance.

2.2 Commodity flow and the entropy cost:

The commodity flow or demand/unit time is of the form

D= -k(P(t)-P0(t))(1)

The concept represented by equation (1) is analogous to energy flow (heat or work) between a thermodynamics system and its environment where k (analogous to a thermal capacity) represents the change in the flow for the change in the price of a commodity and is measured in additional units sold per year per change in unit price e.g. units/year/$.

Let P(t) be the unit price at time t and P0(t) the market equilibrium price at time t, where P(t)<P0(t) for every $t \in [0, T]$. At constant demand rate P(t)=P and P0(t)=P0 noting that when P<P0 , the direction of the commodity flow is from the system to the surroundings. The entropy generation rate must satisfy

$$S = \frac{d\sigma(t)}{dt} = k\left(\frac{P}{P_0} + \frac{P_0}{P} - 2\right)$$

To illustrate, assume that the price of a commodity decreases according to the following relationship

$P(t) = P(0) - \frac{a}{T}t$ where, $a = P(0) - P(T)$ as linear form of price being time dependent.

$E(t) = \frac{D}{\sigma(T)} = \frac{TP_0^2}{a}\ln\left[1 - \frac{aT}{TP(0) - TP_0}\right] - P_0T =$ Entropy cost per cycle.

Case-1:Instantaneous cash-flows (the case of the basic EnOQ model)

The components of total inventory cost of the system per cycle time are as follows:

(a) Ordering cost = $A + Ae^{-rT} + Ae^{-2rT} + ... = \frac{A}{1-e^{-rT}}$

(b) Present value of the purchase cost can be shown as

$$CT\int Ddt + CT\int De^{-rt}dt + CT\int De^{-2rt}dt + ... = \frac{CT^2k}{1-e^{-rT}}\left[\frac{a}{2} + P_0 - p(0)\right]$$

(c) The present value of the out-of-pocket inventory carrying cost can be shown as

$$hc\left[\int_0^T D(T-t)e^{-rt}dt + \int_0^T D(T-t)e^{-r(T+t)}dt + \int_0^T D(T-t)e^{-r(2T+r)}dt + ...\right]$$

$$= \frac{hck}{r^2}\left(P(0) - P_0 - \frac{2a}{rT}\right) + \frac{hcka\left(1+e^{-rT}\right)}{r^2\left(1+e^{-rT}\right)} + \frac{hc}{1+e^{-rT}}\left(\frac{kT}{r}\left(p_0 - P(0)\right)\right)$$

So, the present value of all future cash-flow in this case is

$$PV_1(T) = \frac{A}{1-e^{-rT}} + \frac{cT^2k}{1-e^{-rT}}\left(\frac{a}{2} + p_0 - P(0)\right) + \frac{hck}{r^2}\left(P(0) - P_0 - \frac{2a}{rT}\right)$$

$$+ \frac{hcka\left(1+e^{-rT}\right)}{r^2\left(1-e^{-rT}\right)} + \frac{hc}{1-e^{-rT}}\left(\frac{kT}{r}\left(p_0 - P(0)\right)\right) + \frac{TP_0^2}{a}\ln\left(1 - \frac{a}{p(0) - P_0}\right) - P_0T \quad (1)$$

Case -2:Credit only on units in stock when $T \le M$.

During the credit period M, the firm makes payment to the supplier immediately after the use of the stock. On the last day of the credit period, the firm pays the remaining balance. Furthermore, the credit period is greater than the inventory cycle length. The present value of the purchase cost can be shown as

$$C\left[\int_0^T De^{-rT}dt + \int_0^T De^{-r(T+t)}dt + \int_0^T De^{-r(2T+t)}dt + ...\right]$$

$$= \frac{C}{1-e^{-rT}}\left\{\frac{K(p(0) - P_0)}{r}\left(1-e^{-rT}\right) + \frac{Ka}{T}\left(\frac{1}{r^2}\left(1-e^{-rT}\right) - \frac{Te^{-rT}}{r}\right)\right\}$$

$$= \frac{-ck\left(P(0) - P_0\right)}{r} + \frac{cka}{Rr^2} - \frac{CTe^{-rT}ka}{rt\left(1-e^{-rT}\right)}$$

The present value of all future cash flows in this case is

$$PV_2(T) = \frac{A}{1-e^{-rT}} - \frac{ck\left(P(0) - P_0\right)}{r} + \frac{cka}{Tr^2} - \frac{ckae^{-rT}}{r\left(1-e^{-rT}\right)} + \frac{hck}{r^2}\left[P(0) - P_0 - \frac{2a}{rT}\right]$$

$$+ \frac{hcka\left(1+e^{-rT}\right)}{r^2\left(1-e^{-rT}\right)} + \frac{hc}{1-e^{-rT}}\left(\frac{kT}{r}\left(P_0 - P(0)\right)\right) + \frac{TP_0^2}{a}\ln\left[1 - \frac{aT}{T(P(0) - P_0T}\right] \quad (2)$$

Case-3: Credit only on units in stock when $T \ge M$

The present value only on units in stock can be shown as

$$C\left[\int_0^M De^{-rt}dt + D(T-M)e^{-rM} + \int_0^M De^{-r(T+t)}dt + D(T-M)e^{-r(M+T)} + ...\right]$$
$$= \frac{\left(e^{-rM}-1\right)}{r^2T\left(1-e^{-rT}\right)}\left[rTc\left(kP(0)-P_0k\right)\right]$$
$$-\frac{1}{rT\left(1-e^{-rT}\right)}\left[caMe^{-rM} + rkc(T-M)e^{-rM}\left(TP(0)-at-P_0T\right)\right]$$

Therefore the present value of all future cash-flows in this case is

$$PV_3(T) = \frac{A}{1-e^{-rT}} + \frac{hck}{r^2}\left(P(0)-P_0-\frac{2a}{rT}\right) + \frac{hcka\left(1+e^{-rT}\right)}{r^2\left(1-e^{-rT}\right)} + \frac{hc}{1-e^{-rT}}\left(\frac{kT}{r}\left(P_0-P(0)\right)\right)$$
$$+\frac{\left(e^{-rM}-1\right)}{r^2T\left(1-e^{-rT}\right)}\left[rTc\left(kP(0)-P_0k-cak\right)\right]$$
$$-\frac{1}{rT\left(1-e^{-rT}\right)}\left[caMe^{-rM} + rkc(T-M)e^{-rM}\left(TP(0)-at-P_0T\right)\right]$$
$$+\frac{TP_0^2}{a}\ln\left(1-\frac{aT}{p(0)-P_0}\right) - P_0T \qquad (3)$$

Now our main aim is to minimize the present value of all future cash-flow cost $PV_\infty(T)$. That is

Minimize $PV_\infty(T)$

subject to T>0.

We will discuss the situations of the two cases,

(A) Suppose M>W/D

In this case we have

$$PV_\infty(T) = \begin{cases} PV_1(T) & if \quad 0<T<W/D \\ PV_2(T) & if \quad W/D \le T < M \\ PV_3(T) & if \quad M \le T. \end{cases}$$

It was found that

$PV_1(T) - PV_2(T) > 0$ and $PV_1(T) - PV_3(T) > 0$

for T>0 and $T \ge M$ respectively.

which implies

$PV_1(T) > PV_2(T)$ and $PV_1(T) > PV_3(T)$

for T>0 and $T \ge M$ respectively.

Now we shall determine the optimal replenishment cycle time that minimizes present value of cash-out-flows. The first order necessary condition for $PV_1(T)$ in (1) to be minimized is expressed as

$$\frac{\partial PV_1(T)}{\partial T} = 0$$

which implies

$$-\frac{re^{-rT}A}{\left(1-e^{-rT}\right)^2} + \frac{2CT\left(1-e^{-rT}\right) - re^{-rT}CT^2}{\left(1-e^{-rT}\right)^2}\left(\frac{ak}{2}+P_0k-kP(0)\right)$$
$$+\frac{hcka}{r^2}\left[\frac{-re^{-rT}\left(1-e^{-rT}\right)-\left(1+e^{-rT}\right)re^{-rT}}{\left(1-e^{-rT}\right)^2}\right]$$

$$+\frac{hck}{r}\left(P_0-P(0)\right)\left[\frac{\left(1-e^{-rT}\right)-re^{-rT}T}{\left(1-e^{-rT}\right)^2}\right]+\frac{2hcka}{r^3T^2}+\frac{P_0^2}{a}\ln\left[1-\frac{a}{P(0)-P_0}\right]-p_0=0 \quad (4)$$

Similarly, $PV_2(T)$ in equation(2) to be minimized is $\frac{\partial PV_2(T)}{\partial T}=0$ which implies

$$-\frac{re^{-rT}A}{\left(1-e^{-rT}\right)^2}-\frac{cka}{r}\left[\frac{-re^{-rT}\left(1-e^{-rT}\right)-e^{-2rT}r}{\left(1-e^{-rT}\right)^2}+\frac{hcka}{r^2}\left[\frac{-re^{-rT}\left(1-e^{-rT}\right)-\left(1+e^{-rT}\right)re^{-rT}}{\left(1-e^{-rT}\right)^2}\right]\right]$$

$$+\frac{hck\left(P_0-P(0)\right)}{r}\left[\frac{\left(1-e^{-rT}\right)-rT\left(re^{-rT}\right)}{\left(1-e^{-rT}\right)}\right]+\frac{cka}{r^2T^2}+\frac{2hcka}{r^3T^2}+\frac{P_0^2}{a}\ln\left[1-\frac{a}{P(0)-P_0}\right]-p_0=0 \quad (5)$$

Likewise, the first order necessary condition for $PV_3(T)$ in equation(3) to be minimized is $\frac{\partial PV_3(T)}{\partial T}=0$ which leads

$$-\frac{re^{-rT}A}{\left(1-e^{-rT}\right)^2}-\frac{hcka\left(-re^{-rT}\left(1-e^{-rT}\right)-\left(1+e^{-rT}\right)re^{-rT}\right)}{r^2\left(1-e^{-rT}\right)^2}$$

$$+\frac{hcrk\left(P_0-P(0)\right)\left(1-e^{-rT}-rTe^{-rT}\right)}{r^2\left(1-e^{-rT}\right)}+rc\left(e^{-rM}-1\right)\left(KP(0)-P_0k-ca\right)$$

$$\frac{P_0^2}{a}\ln\left[1-\frac{a}{P(0)-P_0}\right]-p_0+\frac{2hcka}{r^3T^2}=0 \quad (6)$$

Furthermore, we let

$$\Delta_1=\left.\frac{\partial PV_1(T)}{\partial T}\right|_{T=W/D} \quad (7)$$

$$\Delta_2=\left.\frac{\partial PV_2(T)}{\partial T}\right|_{T=W/D} \quad (8)$$

$$\Delta_3=\left.\frac{\partial PV_3(T)}{\partial T}\right|_{T=M} \quad (9)$$

Lemma-1

(a) If $\Delta_1 \geq 0$, then the total present value of PV1(T) has the unique minimum value at the point T=T1 where $T_1 \in (0, W/D)$ and satisfies $\frac{\partial PV_1}{\partial T}=0$.

(b) If $\Delta_1 < 0$, then the value of $T_1 \in (0, W/D)$ which minimizes PV1(T) does not exist.

Proof:

Now taking the second derivative of PV1(T) with respect to $T_1 \in (0, W/D)$, we have

$$\frac{r^2e^{-rT}\left(1-e^{-rT}\right)^2+2r^2e^{-rT}\left(1-e^{-rT}\right)}{\left(1-e^{-rT}\right)^4}$$

$$+\left\{\frac{2c\left(1-e^{-rT}\right)-4rcT}{\left(1-e^{-rT}\right)^2}-\frac{r^2c^2T^2e^{-rT}\left(1+e^{-rT}\right)}{\left(1-e^{-rT}\right)^3}\right\}\left(\frac{a}{2}+P_0k-kP(0)\right)+\frac{hcka}{r^2}\left(\frac{2e^{-rT}}{\left(1-e^{-rT}\right)^2}\right)$$

$$+\frac{2hckae^{-2rT}}{r\left(1-e^{-rT}\right)^2}+\frac{4hcka}{r^3T^3}+\frac{hck(P_0-P(0))}{\left(1-e^{-rT}\right)^2}\left[e^{-rT}(1+rT)-2r\right]+\frac{2r^2hck(P_0-P(0))T.e^{-rT}}{\left(1-e^{-rT}\right)^3}$$

We obtain from the above expression $\frac{\partial^2 PV_1(T)}{\partial T^2}>0$, which implies $\frac{\partial PV_1(T)}{\partial T}>0$ is strictly increasing function of T in the interval (0,W/D).

Also we know that,

$$\lim_{T\to 0}\left(\frac{\partial PV_1(T)}{\partial T}\right)=-rA<0 \quad \text{and} \quad \lim_{T\to W/D}\left(\frac{\partial PV_1(T)}{\partial T}\right)=\Delta_1$$

Therefore, if $\lim_{T\to W/D}\left(\frac{\partial PV_1(T)}{\partial T}\right)=\Delta_1\geq 0$, then by applying the intermediate value theorem, there exists a unique value $T_1\in(0,W/D)$ such that $\frac{\partial PV_1(T)}{\partial T}=0$ and $\frac{\partial^2 PV_1(T)}{\partial T^2}$ at the point T1 is greater than zero.

Thus $T_1\in(0,W/D)$ is the unique minimum solution to $PV_1(T)$.

However, if $\lim_{T\to W/D}\frac{\partial PV_1(T)}{\partial T}=\Delta_1<0$ for all $T\in(0,W/D)$ and also we find $\frac{\partial^2 PV_1(T)}{\partial T^2}<0$ for all $T\in(0,W/D)$. Thus $PV_1(T)$ is a strictly decreasing function of T in the interval $(0,W/D)$. Therefore, we can not find a value of T in the open interval $(0,W/D)$ that minimizes $PV_1(T)$. This completes the proof.

Lemma-2

(a) If $\Delta_2\leq 0\leq\Delta_1$, then the total present value of PV2(T) has the unique minimum value at the point T=T2 where $T_2\in(W/D,M)$ and satisfies $\frac{\partial PV_2}{\partial T}=0$.

(b) If $\Delta_2>0$, then the present value PV2(T) has a minimum value at the lower boundary point $T=W/D$.

(c) If $\Delta_2<0$, then the present value PV2(T) has a minimum value at the upper boundary point T=M.

Proof:

$$\frac{\partial^2 PV_2(T)}{\partial T^2}=\frac{r^2e^{-rT}\left(1-e^{-rT}\right)^2+2r^2e^{-rT}\left(1-e^{-rT}\right)}{\left(1-e^{-rT}\right)^4}$$

$$-\frac{ckare^{-rT}\left(1+e^{-rT}\right)}{\left(1-e^{-rT}\right)^3}+\frac{hckae^{-2rT}}{r\left(1+e^{-rT}\right)^4}+\frac{hck(P_0-P(0))}{\left(1+e^{-rT}\right)^2}\left\{e^{-rT}\left(1+rT\right)-2r\right\}$$
$$+\frac{2hr^2ck((P_0-P(0))Te^{-rT}}{\left(1-e^{-rT}\right)^3}+\frac{2cka}{r^2T^3}-\frac{4hcka}{r^3T^3}$$

which is >0 where $T\in\left[W/D,M\right]$.

Which implies $\frac{\partial PV_2}{\partial T}$ is strictly increasing function of T in the interval $(W/D,M)$.

Also we have $\left.\frac{\partial PV_2(T)}{\partial T}\right|_{T=W/D}=\Delta_2$.

If $\Delta_2\le 0$, then by applying the intermediate value theorem there exists a unique value $T_2\in\left[W/D,M\right]$ so that $\left.\frac{\partial PV_2(T)}{\partial T}\right|_{T=T_2}=0$. Moreover, by taking the second derivative of PV2(T) with respect to T at the point T2 we have $\frac{\partial^2 PV_2}{\partial T^2}>0$.

Thus $T_2\in\left[W/D,M\right]$ is the unique solution to $PV_2(T)$.

Now if $\left.\frac{\partial PV_2(T)}{\partial T}\right|_{T=W/D}=\Delta_2>0$ which implies $\frac{\partial PV_2}{\partial T}>0$ for all $T_2\in\left(W/D,M\right)$. Therefore, $PV_2(T)$ is strictly increasing function of T in the interval $\left(W/D,M\right)$. Therefore $PV_2(T)$ has a minimum value at the lower boundary point $T=W/D$ and similarly we can prove $PV_2(T)$ has a minimum value at upper boundary point T=M.

Lemma-3

(a) If $\Delta_3\le 0$, then the total present value of $PV_3(T)$ has the unique minimum value at the point T=T3 on $T_{31}\in(M,\infty)$ and satisfies $\frac{\partial PV_3}{\partial T}=0$.

(b) If $\Delta_3>0$, then the present value of $PV_3(T)$ has a minimum value at the boundary T=M.

Proof:

The proof is same to the lemma-2.

Proposition 1

(i) $2e^{rt}-2-rT>0$

(ii) $e^{2rT}+1-3e^{rT}>0$

Proof:

(i) $2e^{-rT}-2-rT=2\left(e^{rT}-1\right)-rT$

$$=2\left(1+rt+\frac{(rT)^2}{2!}+.....+1\right)-rT$$

$$=rt+2\left(\frac{(rT)^2}{2!}+\frac{(rT)^3}{3!}+.....\right)$$

which is always +ve as value of r and T are always positive.

(ii) $e^{2rT}+1-3e^{rT}$

$$= 1 + 2rT + \frac{(2rT)^2}{2!} + \frac{(3rT)^3}{3!} + ... + 1 - 3\left(1 + rT + \frac{(rT)^2}{2!} + \frac{(rT)^3}{3!} + ...\right)$$

$$= -1 - rT + \frac{(rT)^2}{2} + \frac{6(rT)^3}{6} + ...$$

$$= -1 - rT + \frac{(rT)^2}{2} + (rT)^3 + 10.5(rT)^4$$

which is also give a positive value a r and T are always positive.

By this two position it is easy to say that $\Delta_1 > \Delta_2$.

Then the equations (7) – (9) yield

$\Delta_1 < 0$ iff $PV_1^/(W/D) < 0$ iff $T_1^* > W/D$ (10)

$\Delta_2 < 0$ iff $PV_2^/(W/D) < 0$ iff $T_2^* > W/D$ (11)

$\Delta_3 < 0$ iff $PV_2^/(M) < 0$ iff $T_2^* > M$ (12)

$\Delta_3 < 0$ iff $PV_3^/(M) < 0$ iff $T_3^* > M$ (13)

From the above equations we have the following results.

Theorem-1

(1) If $\Delta_1 > 0, \Delta_2 \geq 0$ and $\Delta_3 > 0$, then $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(W/D)\}$. Hence T^* is T_1^* or W/D associated with the least cost.

(2) If $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 > 0$, then $PV_\infty(T^*) = PV_\infty(T_2^*)$. Hence T^* is T_2^*.

(3) If $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 \leq 0$, then $PV_\infty(T^*) = PV_\infty(T_3^*)$. Hence T^* is T_3^*.

(4) If $\Delta_1 \leq 0, \Delta_2 < 0$ and $\Delta_3 > 0$, then $PV_\infty(T^*) = PV_\infty(T_2^*)$. Hence T^* is T_2^*.

(5) If $\Delta_1 \leq 0, \Delta_2 < 0$ and $\Delta_3 \leq 0$, then $PV_\infty(T^*) = PV_\infty(T_3^*)$. Hence T^* is T_3^*.

Proof:

(1) If $\Delta_1 > 0, \Delta_2 \geq 0$ and $\Delta_3 > 0$, which imply that $PV_1^/(W/D) > 0$, $PV_2^/(W/D) \geq 0$, $PV_2^/(M) > 0$ and $PV_3^/(M) > 0$. From the above lemma we implies that

(i) $PV_3(T)$ is increasing on $[M, \infty)$

(ii) $PV_2(T)$ is increasing on $[W/D, M)$

(iii) $PV_1(T)$ is increasing on $[T_1^*, W/D)$ and decreasing on $(0, T_1^*]$

Combining above three, we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = W/D$. Hence, $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(W/D)\}$. Consequently, T^* is T_1^* or W/D associated with the least cost.

(2) If $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 > 0$, which imply that $PV_1^/(W/D) > 0$, $PV_2^/(W/D) < 0$, $PV_2^/(M) > 0$ and $PV_3^/(M) > 0$ which implies that $T_1^* \leq W/D$, $T_2^* > W/D$, $T_2^* < M$ and $T_3^* < M$ respectively. Furthermore from the lemma

(i) $PV_3(T)$ is increasing on $[M, \infty)$

(ii) $PV_2(T)$ is decreasing on $[W/D, T_2^*]$ and increasing on $(T_2^*, M]$

(iii) $PV_1(T)$ is decreasing on $(0, T_1^*]$ and increasing on $(T_1^*, W/D]$.

From the above we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = T_2^*$ on $[W/D, \infty)$. Since $PV_1(T) > PV_2(T)$ and T>0. Then $PV_\infty(T^*) = PV_\infty(T_2^*)$ and T^* is T_2^*

(3) If $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 \leq 0$, which implies that $PV_1^/(W/D) > 0$, $PV_2^/(W/D) < 0$, $PV_2^/(M) \leq 0$ and $PV_3^/(M) \leq 0$ which imply that $T_1^* < W/D$, $T_2^* > W/D$, $T_2^* \geq M$ and $T_3^* > M$ respectively. Furthermore, from the lemma it implies that

(i) $PV_3(T)$ is decreasing on (M, T_3^*) and increasing on (T_3^*, ∞)

(ii) $PV_2(T)$ is decreasing on $(W/D, M)$

(iii) $PV_1(T)$ is decreasing on $(0, T_1^*)$ and increasing on $(T_1^*, W/D]$.

Combining all above, we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $[W/D, \infty)$. Since, $PV_2(T)$ is decreasing on $(0, T_2^*)$, $T_1^* < W/D$ and $T_2^* \geq M > W/D$ we have $PV_1(T_1^*) > PV_2(T_2^*)$, $PV_2(T_1^*) > PV_2(M)$ and $PV_3(M) > PV_3(T_3^*)$. Hence we conclude that $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $(0, \infty)$. Consequently, T^* is T_3^*.

(4) If $\Delta_1 \leq 0, \Delta_2 < 0$ and $\Delta_3 > 0$, which implies that $PV_1^/(W/D) \leq 0$, $PV_1^/(W/D) \leq 0$, $PV_2^/(M) > 0$ and $PV_3^/(M) > 0$ which imply that $T_1^* \geq W/D$, $T_2^* > W/D$, $T_2^* < M$ and $T_3^* < M$. Furthermore, we have

(i) $PV_3(T)$ is increasing on $[M, \infty)$

(ii) $PV_2(T)$ is decreasing on $[W/D, T_2^*]$ and increasing on $(T_2^*, M]$

(iii) $PV_1(T)$ is decreasing on $(0, W/D)$.

Since $PV_1(W/D) > PV_2(W/D)$, and $PV_2(W/D) > PV_2(T^*)$

So we conclude that $PV_\infty(T)$ has the minimum value at $T = T_2^*$ on $(0, \infty)$. Consequently, T^* is T_2^*.

(5) If $\Delta_1 \leq 0, \Delta_2 < 0$ and $\Delta_3 \leq 0$, which gives that $PV_1^/(W/D) \leq 0$, $PV_2^/(W/D) < 0$, $PV_2^/(M) \leq 0$ and $PV_3^/(M) \leq 0$ and which imply that $T_1^* \geq W/D$, $T_2^* > W/D$, $T_2^* \geq M$ and $T_3^* \geq M$ respectively. Furthermore, from the lemma it implies that

(i) $PV_3(T)$ is decreasing on (M, T_3^*) and increasing on $[T_3^*, \infty)$

(ii) $PV_2(T)$ is decreasing on $[W/D, M]$

(iii) $PV_1(T)$ is decreasing on $(0, W/D)$.

Since $PV_1(W/D) > PV_2(W/D)$, combining the above we conclude that $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $(0, \infty)$. Consequently, T^* is T_3^*. This completes the proof.

(B) Suppose $M \le W/D$.

Here $PV_\infty(T)$ can be expressed as follows:

$$PV_\infty(T) = \begin{cases} PV_1(T) & if \quad 0 < T < W/D \\ PV_2(T) & if \quad W/D \le T \end{cases}$$

and let $\left.\frac{\partial P_3^/(T)}{\partial T}\right|_{T=W/D} = \Delta_4$ (14)

By using proposition 1, we have

$\Delta_1 - \Delta_4 > 0$, which leads to $\Delta_1 > \Delta_4$.

From (14), we also find that

$\Delta_4 < 0$ iff $PV_3^/(W/D) < 0$ iff $T_3^* > W/D$ (15)

Lemma-4

(a) If $\Delta_4 \le 0$, then the present value of $PV_3(T)$ possesses the unique minimum value at the point $T = T_3$, where $T_3 \in [W/D, \infty)$ and satisfies $\frac{\partial P_3\,(T)}{\partial T} = 0$.

(b) If $\Delta_4 > 0$, then the present value of $PV_3(T)$ possesses a minimum value at the boundary point $T = W/D$.

Proof: The proof is similar to that of Lemma-2.

Theorem-2

(1) If $\Delta_1 > 0$ and $\Delta_4 \ge 0$, then $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(W/D)\}$. Hence T^* is T_1^* or W/D is associated with the least cost.

(2) If $\Delta_1 > 0$ and $\Delta_4 < 0$, then $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(T_3^*)\}$. Hence T^* is T_1^* or T_3^* is associated with the least cost.

(3) If $\Delta_1 \le 0$ and $\Delta_4 < 0$, then $PV_\infty(T^*) = PV_\infty(T_3^*)$. Hence T^* is T_3^*.

Proof:

(1) If $\Delta_1 > 0$ and $\Delta_4 > 0$, which imply that $PV_1^/(W/D) > 0$ and $PV_3^/(W/D) \ge 0$, and also $T_1^* < W/D$ and $T_3^* \le W/D$. Furthermore, we have

(i) $PV_3(T)$ is increasing on $[W/D, \infty)$

(ii) $PV_1(T)$ is decreasing on $(0, T_1^*]$ and increasing on $[T_1^* < W/D)$. Combining the above we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = W/D$ on $[W/D, \infty)$. Hence, $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(W/D)\}$. Consequently, T^* is T_1^* or W/D associated with the least cost.

(2) If $\Delta_1 > 0$ and $\Delta_4 < 0$, which imply that $PV_1^{'}(W/D) > 0$ and $PV_3^{/}(M) < 0$ which implies that $T_1^* < W/D$ and $T_3^* > W/D$ and also

(i) $PV_3(T)$ is decreasing on $[W/D, T_3^*]$ and increasing on $[T_3^*, \infty)$

(ii) $PV_1(T)$ is decreasing on $(0, T_1^*]$ and increasing on $(T_1^*, W/D]$. Combining (i) and (ii) we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $[W/D, \infty)$. Hence, $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(T_3^*)\}$. Consequently, T^* is T_1^* or T_3^* associated with the least cost.

(3) If $\Delta_1 \le 0$ and $\Delta_4 \le 0$, which implies that $PV_1^{/}(W/D) \le 0$ and $PV_3^{/}(M) < 0$ and $T_1^* \ge W/D$ and $T_3^* > W/D$ and also

(i) $PV_3(T)$ is decreasing on. $[W/D, T_3^*]$ and increasing on $[T_3^*, \infty)$.

(ii) $PV_1(T)$ is decreasing on $(0, W/D)$

From which we conclude that $PV_\infty(T)$ is decreasing on $(0, W/D)$ and $PV_\infty(T)$ hs the minimum value at $T = T_3^*$ on $[W/D, \infty)$. Since, $PV_1(W/D) > PV_3(W/D)$, and conclude that $PV_\infty(T)$ has minimum value at $T = T_3^*$ on $(0, \infty)$. Consequently T^* is T_3^*.
This completes the proof.

NUMERICAL EXAMPLES

The followings are considered to be its base parameters A=$5/order, r=0.3/$, C=$1. a=1, k=2.4, P0(Market Price)=$3, Price at the beginning of a cycle P(0)=$1, D=-k(P(0)-P0)=-2.4(1-3)=4.8

Example-1

If M=2, W=2, W/D<M

Δ_1=31.47>0, Δ_2=-2.5020<0, Δ_3=-1.520559<0

T^*=T3=2.55, $PV_3(T)$=36.910259

Example-2

If M=2, W=3, W/D<M

Δ_1=48.416874>0, Δ_2=-0.112<0, Δ_3=-1.52<0

T^*=T3=2.55, $PV_3(T)$=36.910259

Example-3

If M=5, W=3, W/D<M

Δ_1=48.51>0, Δ_2=-0.112<0, Δ_3=2.1715>0

T^*=T2=3.1, $PV_2(T)$=36.302795

Example-4

If M=20, W=6, W/D<M

Δ_1=83.5186>0, Δ_2=1.47>0, Δ_3=4.001>0

T^*=T_1^* =0.7442, $PV_1(T)$=48.134529

Example-5

If M=5, W=2, W/D<M

Δ_1=36.5465>0, Δ_2=-2.5<0, Δ_3=2.738>0

$T^* = T_2^*$ =3.1, $PV_2(T)$ =36.302795

Example-6

If M=2, W=1, W/D<M

Δ_1=-1.546<0, Δ_2=-15.079<0, Δ_3=-1.52<0

$T^* = T_3^*$ =2.55, $PV_3(T)$ =36.910259

Example-7

If M=10, W=1, W/D<M

Δ_1=-1.546<0, Δ_4=-15.079<0, Δ_3=3.728554>0

$T^* = T_2^*$ =3.1, $PV_2(T)$ =36.302795

Example-8

If M=1, W=5, W/D<M

Δ_1=164..83>0, Δ_4=2.946>0

$T^* = T_1$ =0.7442, $PV_1(T)$ =48.134529

Example-9

If M=10, W=5, W/D<M

Δ_1=83.518661>0, Δ_4=-1.837413<0

$T^* = T_3^*$ =2.66, $PV_3(T)$ =34.98682

Based on the above computational result of the numerical examples, the following managerial insights are obtained and following comparative evaluation are observed. If the supplier does not allow the delay payment, cash-out-flow is more but practically taking in view of real-world market, to attract the retailer(customer)credit period should be given and it observed that it should be less than equal to the inventory cycle to achieve the better goal. Furthermore, it is preferable for the supplier to opt a credit period which is marginally small.

CONCLUSION AND FUTURE RESEARCH

This paper suggested that it might be possible to improve the performance of a market system by applying the laws of thermodynamics to reduce system entropy (or disorder). It postulates that the behaviours of market systems very much resembles that of physical system operating within surroundings, which include the market and supply system.

In this paper, the suggested demand is price dependent. Many researchers advocated that the proper estimation of input parameters in EOQ models which is essential to produce reliable results. However, some of those costs may be difficult to quantify. To address such a problem, we propose in this paper accounting for an additional cost (entropy cost) when analysing EOQ systems which allow a permissible delay payments if the retail orders more than or equal to a predetermined quantity. The results from this paper suggest that the optimal cycle time is more sensitive to the change in the quantity at which the fully delay payment is permitted.

An immediate extension is to investigate the proposed model to determine a retailer optimal cycle time and the optimal payment policy when the supplier offers partially or fully permissible delay in payment linking to payment time instead of order quantity.

ACKNOWLEDGEMENTS

The authors sincerely acknowledge the anonymous referees for their constructive suggestion.

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