Image Denoising Using Earth Mover's Distance and Local Histograms

Nezamoddin N. Kachouie nezam.nk@gmail.com
Department of Systems Design Engineering,
University of Waterloo,Waterloo, ON, Canada
Present Affiliation: Harvard-MIT Health Sciences and Technology
Harvard Medical School,Cambridge,MA,USA

Abstract

In this paper an adaptive range and domain filtering is presented. In the proposed method local histograms are computed to tune the range and domain extensions of bilateral filter. Noise histogram is estimated to measure the noise level at each pixel in the noisy image. The extensions of range and domain filters are determined based on pixel noise level. Experimental results show that the proposed method effectively removes the noise while preserves the details. The proposed method performs better than bilateral filter and restored test images have higher signal to noise ratio than those obtained by applying popular Bayesshrink wavelet denoising method.

Keywords: Denoising, Bilateral filtering, Local histogram, Earth mover's distance.

1. INTRODUCTION

Noise elimination is an important concern in image processing and computer vision. Images obtained from the real world are corrupted with noise. The image noise might decrease to some negligible levels under ideal conditions such that denoising is not necessary, but in general to recover the image the corrupting noise must be removed for practical purposes. Noise makes ambiguities in the underlying signal relative to its observed form by perturbations which are not related to the scene under study. The goal of denoising is to remove the noise and to retain the important signal features as much as possible. Linear filters, which consist of convolving the image with a constant matrix to obtain a linear combination of neighborhood values, have been widely used for noise elimination in the presence of additive noise. However they can produce a blurred and smoothed image with poor feature localization and incomplete noise suppression.

Gaussian filters are typical linear filters that have been widely used for image denoising. Gaussian filters assume that image signals have smooth spatial variations and pixels in a neighborhood have close values, so noise will be suppressed while signal will be preserved by averaging pixel values over a local neighborhood. The assumption of slow spatial signal variations works well in smooth regions; however it fails and undesirably blurs the signal where spatial variations are high such as edges.

To overcome this shortcoming and prevent undesirable blurring in regions with high spatial signal variations, a number of filters in spatial and spatial-frequency domain are proposed. The most popular ones in spatial domain are anisotropic diffusion [1-3], bilateral filtering and its extensions [4-8]. Diffusion based methods iteratively solve partial differential equations and average the signal over spatial neighborhood whose extension is determined based on local signal variations. Bilateral filtering also known as range and domain filtering is a non-linear filter which performs weighted averaging in both range and domain.

Bilateral filtering was introduced by Tomasi and Manduchi [4] to smooth noisy images while preserve edges using neighboring pixels. Bilateral filtering is a local, nonlinear, and noniterative technique which considers both gray level (color) similarities and geometric closeness of the neighboring pixels. In a traditional domain filter, weight of the pixels decays by distance from the center of the filter. Low pass filters assume that spatial variations is slow over the image, so by weighted averaging of pixel values in a neighborhood, noise will be averaged away while the signal will be preserved. However, this assumption fails at edges where the spatial variations are not smooth and application of the low pass filter blurs the edges. Bilateral filter overcomes this by filtering the image in both range and domain. Pixels in a neighborhood are considered close either based on their spatial location (domain), or based on their pixel values (range). Therefore bilateral filter averages pixel values based on weights that decay by both distance and pixel dissimilarity.

There are several extensions to improve bilateral filtering [5-8]. In [6], a training-based bilateral filtering is proposed where a general degradation model is considered for degraded images. Then a restoration algorithm is developed to restore the degraded images with unknown degradation process. Therefore, the success of restoration process depends on the general definition of degradation model. In [7,8], different methods to speed up the bilateral filtering have been proposed.

Nonlinear filters in spatial-frequency domain have also been proposed to preserve detail signal and suppress the noise. The most popular ones are wavelet based denoising techniques [9-12]. In wavelet based denoising methods, the noise is estimated and wavelet coefficients are thresholded to separate signal and noise. Various approaches to nonlinear wavelet-based denoising have been introduced among them Bayesshrink wavelet denoising is developed in the Bayesian framework and has been widely used for image denoising [10-11].

In this paper, an adaptive technique is proposed to tune the extensions of range and domain filters. In the proposed method, the distance of the local histogram from the estimated noise histogram is measured using earth mover's distance. The measured distance at each spatial location is then used for adaptive tuning of bilateral filter. The proposed method provides promising results and effectively removes the noise while preserves the signal characteristics. The proposed method is presented in the next section followed by results and conclusions.

2. The Proposed Method

Let pure signal *S* (here an image) be distorted by additive noise *n*. We can write

$$I = S + n \cdot 1 \tag{1}$$

where *I* is the noisy signal. The goal of denoising is separating signal *S* and noise *n* by estimating *n* such that *S* can be extracted from *I*:

$$\hat{S} = I - \hat{n} \cdot 1 \tag{2}$$

This can be done by applying a filter *h* to the signal *I*

$$\hat{S} = h * I \tag{3}$$

where traditionally h is defined as a local filter assigning higher weights to neighboring pixels which are spatially closer to the central pixel x_c of the neighborhood. A popular and simple case of h is Gaussian filter

$$h = h_d(x, \mu_d, \sigma_d) = \exp\left\{-\frac{1}{2}\left(\frac{\|x-\mu_d\|}{\sigma_d}\right)^2\right\} \tag{4}$$

where $\mu_d = x_c$ is the central pixel of the neighborhood such that $d(x, \mu_d) = |x - \mu_d|$ is the Euclidean distance between x_c and a neighboring pixel x. Gaussian domain filtering by using a Gaussian filter averages away noise and preserves the signal in smooth regions, however in the same way it averages away and blurs signal details such as edges. A popular solution to solve this problem is employing bilateral filter [4].

Bilateral Filter

Bilateral filter combines range and domain filtering

$$h(x, \mu_d, \sigma_d, \mu_r, \sigma_r) = h_d(x, \mu_d, \sigma_d) h_r(x, \mu_r, \sigma_r) \tag{5}$$

where the range filter averages the signal values in a neighborhood by assigning the weights based on the similarity of the neighboring pixels and the central pixel:

$$h_r(x, \mu_r, \sigma_r) = \exp\left\{-\frac{1}{2}\left(\frac{\|I(x)-\mu_r\|}{\sigma_r}\right)^2\right\} \tag{6}$$

where $\mu_r = I(\mu_d) = I(x_c)$ is the intensity value of the central pixel of the neighborhood such that $r(I(x), \mu_r) = |I(x) - \mu_r|$ is the absolute intensity difference of the central and a neighboring pixel x.

Bilateral filtering overcomes the shortcomings of linear domain filtering by combining the linear domain filter with a nonlinear range filter. As a result bilateral filter preserves signal details such as edges while suppresses noise, however it considers fix parameters (σ_d, σ_r) for extensions of both domain and range filters. The performance of bilateral filter can be improved by adaptively tuning the filter parameters over the image based on the spatial noise level.

Adaptive Range and Domain Filtering

In the proposed method, to improve the performance of bilateral filtering, spatial noise level is locally estimated to determine the filter parameters (σ_d, σ_r).

To estimate the local spatial noise level n_l, the image noise histogram n_g is estimated and compared with the local signal. To compare two probability density functions (PDFs), a number of nonparametric models have been used including minimizing the comparison $\aleph^2$ function between two PDFs.

The $\aleph^2$ distance between histograms of two delta functions $\delta(x1)$ and $\delta(x2)$ where $x_1 \neq x_2$ is the same regardless of the distance between x_1 and x_2. This is not generally suitable for many image processing applications where different smooth regions could be represented with disjoint δ functions.

The earth mover's distance (EMD) or the Wasserstein distance is a mathematical measure to compare distributions (histograms). EMD was first introduced by Gaspard Monge in 1781, it was later used as a distance measure for intensity images [13]. The EMD between two distributions is the least work that is required to move one distribution to another such that two distributions completely cover each other.

Let H_a and H_b be two normalized histograms with cumulative distributions C_a and C_b respectively. EMD between H_a and H_b is defined by

$$E(H_a, H_b) = \int_0^1 |C_a(k) - C_b(k)| dk \tag{7}$$

Local histogram for each pixel *x* in image *I* is computed over the neighborhood *w* consisting pixel *x* and its neighboring pixels. The EMD is then computed to compare the normalized local histogram H_x and image noise histogram n_g

$$E(H_x, n_g) = \int_0^1 |C_x(k) - C_n(k)| dk \tag{8}$$

where C_x and C_n are cumulative distributions of H_x and n_g respectively. The extensions of domain and range filters (σ_d and σ_r) at each pixel *x* are set using $E(H_x, n_g)$. The domain filter extension at pixel *x* is defined as

$$\sigma_d = \frac{(1 - E(H_x, n_g) + \epsilon_d)}{\sigma} \tag{9}$$

and we have

$$h_d(x, \mu_d, \sigma_d) = \exp\left\{-\frac{1}{2}\left(\frac{\|x - \mu_d\|}{\frac{(1 - \left[\int_0^1 |C_x(k) - C_n(k)| dk\right] + \epsilon_d)}{\sigma}}\right)^2\right\} \tag{10}$$

where $E(H_x, n_g)$ is normalized EMD between noise and pixel histograms, σ is the filter extension parameter, and $\square_d$ is considered to avoid domain filter extension σ_d to be set to zero. The range filter extension also is tuned based on $E(H_x, n_g)$

$$\sigma_r = (1 - E(H_x, n_g) + \epsilon_r) \cdot \sigma \tag{11}$$

and we have

$$h_r(x, \mu_r, \sigma_r) = \exp\left\{-\frac{1}{2}\left(\frac{\|I(x) - \mu_r\|}{(1 - \left[\int_0^1 |C_x(k) - C_n(k)| dk\right] + \epsilon_r) \cdot \sigma}\right)^2\right\} \tag{12}$$

where $\square_r$ is considered to avoid range filter extension σ_r to be set to zero.

Image	Bilateral	Bayesshrink	The Proposed Method
Lena	28.70	29.06	30.09
Boat	27.86	27.87	28.30
Cameraman	29.56	28.63	30.03
Barbara	27.31	26.75	27.39
Goldhill	27.81	28.11	28.56

TABLE 1: Comparison of the proposed method with bilateral and Bayesshrink wavelet filtering methods.

Clearly there is a tradeoff here to choose the domain filter extension σ_d: as the filter extension σ_d expands the number of neighborhood elements grows, allowing for greater noise reduction in the computation but at the same time causing greater spatial blurring by fusion of values from more distant locations. Moreover, the range filter essentially compresses the image histogram by fusion of pixel values and is set by σ_r.

In the proposed method the maximum of σ_d and σ_r are set by σ based on equations (9) and (11). As $0 \le (1 - E) \le 1$, for pixels which are contaminated with high noise, the distance between noise and the pixel histograms E is small, therefore $(1 - E)$ will be large. Considering σ is fixed, σ_d will be large allowing the neighborhood to be extended for greater noise reduction while σ_r will be large based on (11) to allow significant histogram compression.

On the other hand for pixels which are contaminated with low noise, E is large, therefore $(1 - E)$ is small, and in turn σ_d will be small avoiding the neighborhood to be extended which in turn it allows less blurring. Considering that the pixel either is not contaminated with noise or is contaminated with low noise, σ_d will be small. Pixels have close values in small neighborhood, therefore σ_r will be small avoiding significant histogram compression.

Noise Histogram Estimation

To estimate the noise histogram (n_g), the local variance is first computed. Considering the local neighborhood w, the local variance of pixel x is defined as:

$$\sigma_L^2(x) = \frac{\sum_w (I(x_w^i) - \mu_L(x))^2}{N_w} \tag{13}$$

where N_w is the number of pixels in the neighborhood w and

$$\mu_L(x) = \frac{\sum_w I(x_w^i)}{N_w} \tag{14}$$

Noise histogram n_g is estimated by computing the histogram of the local variance image. Further, we set $\sigma = \sigma_n$ where the noise power σ_n^2 is estimated by obtaining the mean of local variance image:

$$\sigma_n^2 = \frac{1}{M \cdot N} \sum_M \sum_N \sigma_L^2(x_{mn}) \tag{15}$$

Finally, the local histogram H_x is computed for each pixel x and EMD is used to measure the distance between noise histogram n_g and local histogram H_x. The schematic of noise histogram estimation is depicted in Fig. 1.

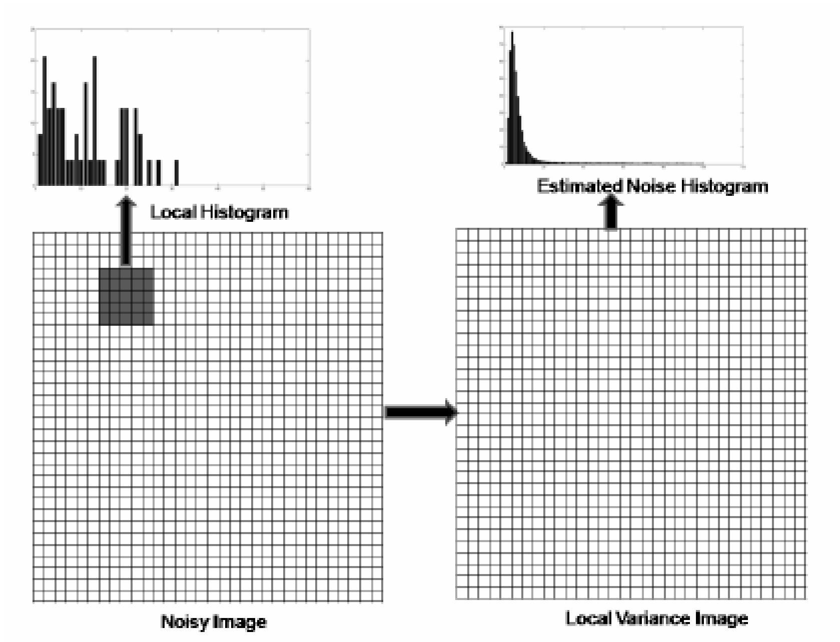

Figure 1: Noise histogram estimation

3. Results and CONSLUSION

To test the proposed method five test images were used. Test images were corrupted by additive Gaussian noise with standard deviation of 15 and 25. The proposed method, the original bilateral filter, and a popular wavelet denoising method so called Bayesshrink wavelet denoising were applied to the corrupted test images. The recovered images applying aforementioned three methods were compared both based on PSNR and visual quality. The results are summarized in Tab. 1.

As we can observe in Tab. 1 for additive Gaussian noise with standard deviation of 15, the proposed method performs better than the original bilateral filtering method. It gains higher PSNR than both the original bilateral filtering and Bayesshrink wavelet denoising methods for all of the test images. The recovered images applying the proposed method have also better visual quality.

The recovered images applying Bayesshrink wavelet and the proposed method are depicted in Fig. 2. The proposed method performs better and the restored image gains higher PSNR (Tab. 1). It has also a better visual quality than that of Bayesshrink method which can be observed by a closer look. Fig. 3 shows the Boat noisy image and EMD computed for the noisy image. The denoised Boat image using the original bilateral filtering, Bayesshrink wavelet, and the proposed method are depicted in this figure.

Figure 2: Restored Lena test image: (a) Bayesshrink wavelet. (b) The proposed method.

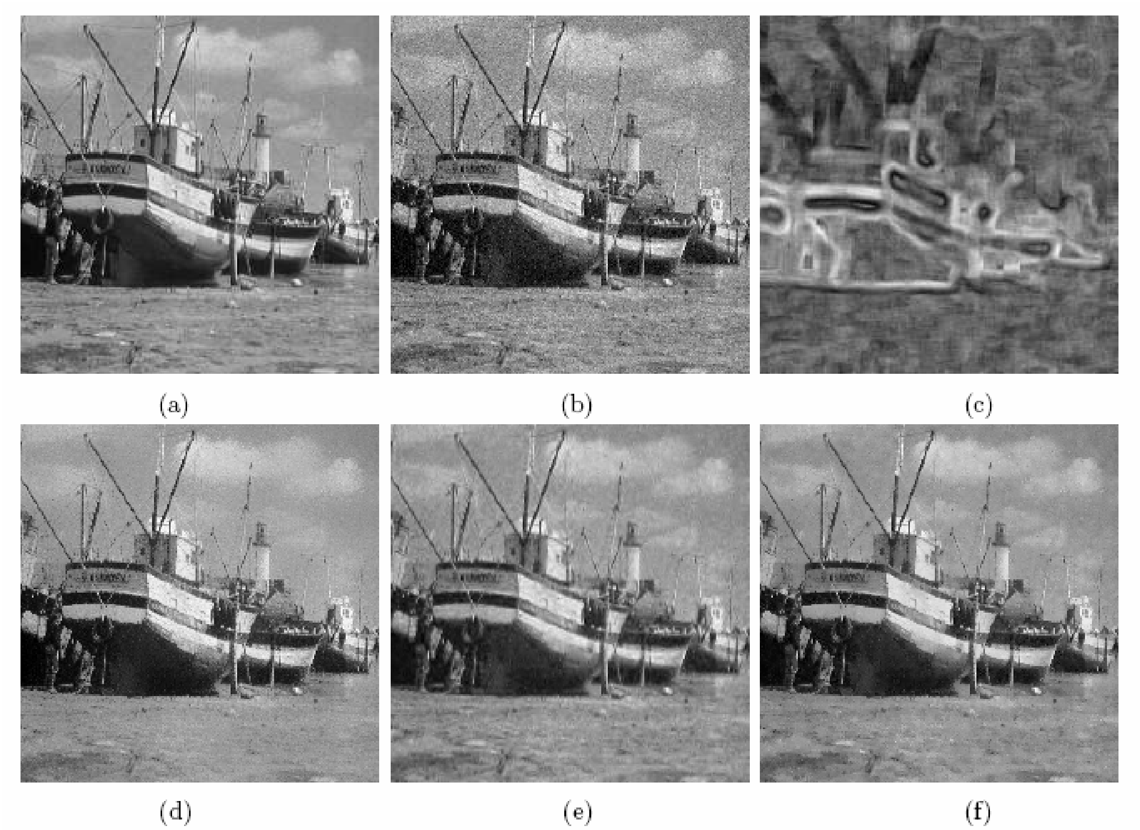

Figure 3: Boat test image: (a) Original image. (b) Noisy. (c) EMD computed for (b). (d) Bilateral. (e) Bayesshrink wavelet. (f) The proposed method.

Fig. 4 shows the application of bilateral filter and the proposed method to the Cameraman test image where the image is corrupted with additive Gaussian noise with σ_n = 25. The application of Bilateral filtering and the proposed method to Goldhill test image which is corrupted with additive Gaussian noise with σ_n = 15 is depicted in Fig. 5. Fig. 6 shows the comparison of the Bayesshrink, bilateral, and the proposed method where they are applied to the Lena test image corrupted with additive Gaussian noise with σ_n = 15. As we can observe in Fig. 4-6, the proposed method performs better and produces smoother results while the details are better preserved in comparison with bilateral filter and the Bayesshrink. It also gains higher PSNR (Tab. 1).

In this paper an adaptive range and domain filtering method based on local histograms was introduced. The noise histogram is estimated and the extensions of range and domain filters are tuned at each spatial location by measuring the distance between the pixel's and noise histograms using earth mover's distance. The proposed method was applied to several test images and its performance was compared with the original bilateral filtering and Bayesshrink wavelet denoising methods. The experimental results obtained by the proposed method showed the improvement of the visual image quality and increase of PSNR in comparison with the bilateral filtering and Bayesshrink wavelet.

Figure 4: Cameraman test image: (a) Original image. (b) Noisy image (PSNR = 20.65). (c) Bilateral filtering (PSNR = 24.70). (d) The proposed method (PSNR = 26.00).

Figure 5: Goldhill test image: (a) Original image. (b) Noisy image (PSNR = 24.71). (c) Bilateral filtering (PSNR = 27.81). (d) The proposed method (PSNR = 28.56).

Figure 6: Lena test image: (a) Original image. (b) Bayesshrink (PSNR = 29.06).
(c) Bilateral filtering (PSNR = 28.70). (d) The proposed method (PSNR = 30.09).

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